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EDITORIAL

" The First Law of Thermodynamics states that heat is energy, and it is therefore proper that it should be measured in the same units as energy. Hitherto the unit has been the calorie, which was chosen rather arbitrarily more than a century ago for its practical convenience, because the early work in calorimetry was mostly done by observing the change of temperature of a water bath. Thus the calorie involved the idea of temperature which is quite irrelevant to the First Law, and it also had reference to an arbitrarily chosen specific substance, water. To use the calorie it was thus necessary to have a number, the mechanical equivalent of heat, which masqueraded as a universal physical constant, though it fact it is nothing but the specific heat of water measured in energy units."

We take the above from a *Memorandum on the unit of heat* prepared by Sir Charles Darwin, F.R.S., Chairman of the Symbols Committee of the Royal Society. The memorandum continues by pointing out the confusion over commonly used calories — at least five definitions are recognised, differing among themselves by 2 parts in 420 — and gives reasons why practical convenience, other than a change of habit for some, is to be identified with thermodynamic logic in this matter. The practical unit of energy, the joule (1 joule = 10^7 erg), having been accepted, by those international bodies most concerned, as the appropriate basic unit of heat, the memorandum recommends its adoption in all numerical work involving thermodynamic quantities in published papers.

There will be many who, by long usage, have " a feel " for the calorie rather than the joule and it is important that their reading should not be hindered by seeing quantities only in unfamiliar form. Consequently the Council of the Royal Society in accepting the logic of its Symbols Committee has recommended that quantities of heat and all other dependent concepts such as specific heat, latent heat, entropy, *etc.* should in future be expressed both in joules and calories. This recommendation has been communicated to the scientific societies in this country, our own included, and was sympathetically received by our Council.

There are few parts of meteorology in which uncertainties of 2 parts in 420 in its measured quantities are of any concern and no one supposes that its progress will be much affected by the adoption of a more rational system of units than has been in common use heretofore. Nevertheless, granted the small effort of adjustment required by those brought up on the calorie, there is some gain in the consistent use of the joule, in simplicity, uniformity with other physical literature, and in logic

itself. Moreover, while first thoughts may suggest the calorie as a very practical unit for meteorology, concerned so much as it is with water, second thoughts point to the joule as the far more practical unit. For meteorology is even more concerned with air than water and the specific heat of air at constant pressure is 1 joule gm^{-1} $°C^{-1}$ to an accuracy of 3 or 4 parts in 1,000. Again, in our formal thermodynamic treatment of the subject, how convenient and error-avoiding is the elimination of that otherwise ubiquitous J or A.

So we ask intending authors of papers for the *Quarterly Journal* to bear in mind this recommendation of the Royal Society, to give a calorie equivalent at least once when joules are quoted; and may the calorie-minded reader find ultimate satisfaction in the joule.

551.521 : 551.510.5

Radiation in the troposphere and lower stratosphere

By R. M. GOODY and G. D. ROBINSON
St. John's College, Cambridge ; Kew Observatory

1. Introduction

1.1. *Scope of the review*

It is only through the absorption of solar radiation and the emission to space the low-temperature terrestrial radiation from the earth and atmosphere that ır planet can receive and lose energy. These two radiative processes are the ɔurce and sink of the irreversible heat engine which is our atmosphere, and before is heat engine is even partly understood we must know the factors which influence ıe emission and absorption of radiation.

The nature of radiative absorption and emission is well understood ; in the reparation of this review therefore it seemed reasonable that only work capable of hysical interpretation should be discussed and complete empiricism avoided. In ıe subsequent paragraphs seven topics, covering every aspect of the subject in hich progress is being made, are treated. The scope of the review has further been stricted by considering only matters of " meteorological importance," by which meant matters related to energy transfer in the atmosphere. Such aspects as ngular distribution and polarisation of scattered radiation, which have been the ıe subject of considerable attention, will be almost completely ignored, as will hotochemical and biological problems.

1.2. *The nature of the problem*

The earth receives energy at a definite rate from the sun, and in turn radiates ɔ space. Its heat content does not change appreciably, so that an overall balance xists. Solar radiation is nearly parallel on arrival at the upper limits of the atmos-here but scattering in the atmosphere attenuates the beam and, with diffuse reflec-on at the surface, adds a non-isotropic field of diffuse radiation. The intensity of ıe incident solar beam, corrected to the mean solar distance — the solar constant — accepted as about 2 cal cm^{-2} min^{-1}. The mean flux of solar energy perpendicular ɔ the earth's surface is about 0·5 cal cm^{-2} min^{-1}. Of this approximately 35 per cent. reflected from the atmosphere (including clouds), 50 per cent. reaches the earth's ırface directly or after scattering, and 45 per cent. is absorbed at the surface. The ıtio of the outward to the inward flux, in a given frequency range, is known as the bedo, and we speak of the albedo of the earth as a whole, and of different surfaces ıch as cloud, snow, sea, or grassland. The accepted value of the albedo of the earth about 0·4, so that an average of about 0·3 cal cm^{-2} min^{-1} is available for heating, irectly and indirectly, the earth and the atmosphere. This energy is distributed by

hydrodynamic and radiative transfer, and returned to space as terrestrial radiation.

The equilibrium temperature of a body in the position of the earth with albedo 0·4 for solar radiation but black for longer wavelength radiation is about 245°K. This sets the general level of terrestrial temperatures. At these temperatures radiation of wavelength less than about 4 μ is of negligible energy, and since the solar flux carries very little energy at wavelengths greater than this it is possible and convenient to treat the solar and terrestrial fluxes quite separately. The earth's surface, cloud, and dust particles radiate very nearly as black bodies ; the principal gaseous constituents of the atmosphere are transparent to their radiation. Minor constituents, such as, H_2O, CO_2, O_3, N_2O, *etc.*, have intense and complex absorption spectra in this region, and are present in sufficient quantity to absorb, and therefore to emit, a considerable proportion of the terrestrial radiation, which is therefore passed out through the atmosphere from one layer of gas to another. This radiative energy transfer offers a complex mathematical problem which at present can only be tackled by numerical or graphical methods. The nature of these absorptions by minor atmospheric constituents is very complex, and much effort has been expended in the laboratory in attempts to obtain the data required for application to atmospheric problems. Owing to the minute quantities of these gases in the atmosphere it has been found in many cases that their most measurable features are the absorption bands shown in the solar spectrum and therefore infra-red solar spectroscopy must often be used to detect and to measure their distribution.

An attempt has been made in Fig. 1 to give a general picture of the relative importance of various absorptions. This figure should be taken only as a very rough guide since ozone and water-vapour absorptions vary considerably from time to time. The absorption spectra apply to the diffuse terrestrial radiation and to a solar beam with a zenith angle of 50°. An indication of the energy absorbed by the stratosphere or troposphere may be obtained by multiplication of (*a*) and (*c*), or (*a*) and [(*b*) — (*c*)] respectively.

In the stratosphere, the absorption of energy from the solar beam can be seen to be controlled mainly by the Hartley band of ozone, while in the troposphere the near infra-red bands of water vapour (ρ, ϕ, ψ, Ω, and χ) are probably of the greatest importance. For the terrestrial stream, water vapour is clearly the most important single constituent of the troposphere, although the 15 μ carbon-dioxide band will also have some influence. In the stratosphere on the other hand, water vapour, carbon dioxide and ozone are likely to be of comparable importance.

Three of the principal atmospheric radiators, cloud, water vapour and ozone, are present in widely variable concentrations, and this makes any general mathematical treatment of radiation quite impracticable. In these circumstances studies of radiation in meteorology have fallen into either of two classes, the one synthetic but limited in scope, the other descriptive but aiming at completeness. The former, typified by the early work of Gold and Emden, approaches from the standpoint of local radiative equilibrium in a standardised cloudless atmosphere, and enquires how far the structure of the real atmosphere can be explained in this way. The other, of which Simpson's work forms the best known example, accepts the observed atmospheric structure, and attempts to work out the resulting field of radiation. The methods are complementary : each throws some light on the nature of the

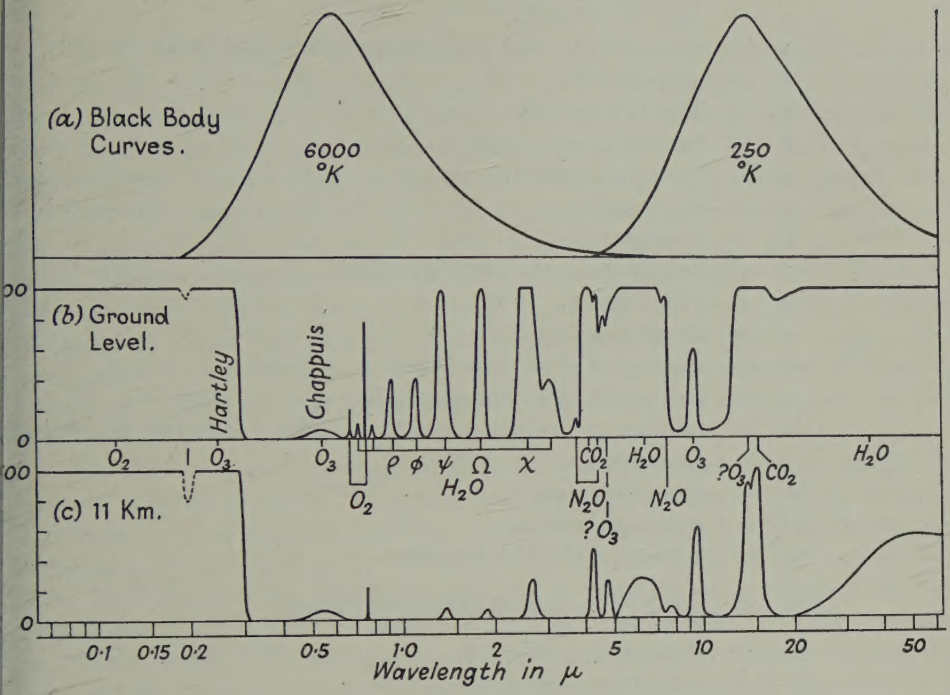

Figure 1. (a) Black-body emission for 6,000°K and 250°K.
(b) Atmospheric absorption spectrum for a solar beam reaching ground level.
(c) The same for a beam reaching the temperate tropopause.

logarithmic wave length scale has been used. Since the solar and terrestrial fluxes must be equal a summation is made over all angles and over the earth's surface, the two curves in (a) have been awn with equal areas. In (b) and (c) no fine structure is shown although all the infra-red bands ive in fact a very complex structure. Most of the bands shown have a broad doublet structure, ut even this is smoothed out. It has been assumed that the atmosphere contains 3 mm of ozone at T.P. all above 11 km ; that it contains 2 gm cm^{-2} of water vapour above ground level and $^{-3}$ gm cm^{-2} above 11 km ; that carbon dioxide and nitrous oxide are mixed in equal proportions ith the atmosphere at all heights. The small window (dotted) at 0.2 μ and the two ozone bands marked (?) have not yet been observed at the two levels concerned.

ydrodynamic processes, the former by the ways in which the deduced structure iffers from that observed, the latter by the computed departures from local radiative alance.

Development of the subject has been retarded not only by the paucity of field bservations, but even more by ignorance of fundamental physical facts. Recent rogress has been the consequence mainly of partial removal of the latter disability. Iany of the remaining questions are more easily soluble in the field than in the aboratory, and there is a pressing need for more observations of radiation in the tmosphere, particularly away from the surface.

1.3. *Symbols*

The following symbols are used in this review :

c = velocity of light.
ν = frequency (or wave number, in accordance with context).
λ = wavelength.
h = Planck's constant.
k = Boltzman's constant.
σ = Stefan's constant.
p = air pressure.
T = temperature.
z = height above the earth's surface.
v = visual range.
r = droplet radius.
δ = spacing of absorption lines.
α = half width of an absorption line.
N = number of molecules cm^{-3} ; a suffix i indicates the ith class of molecules.
M = molecular mass.
D = collision diameter.
ρ = density of absorbing matter.
m = $\int \rho dz$ = amount of absorbing matter.
k_ν = absorption coefficient.
β_ν = extinction coefficient.
k = $\int_0^\infty k_\nu d\nu$ = line or band intensity depending upon context.
$T_\nu(m)$ = transmissivity = $\exp(-k_\nu m)$ or $\exp(-\beta_\nu m)$ by Beer's Law.
$A_\nu(m)$ = absorptivity = $1 - \exp(-k_\nu m)$.
$E_\nu(m)$ = emissivity = $A_\nu(m)$ by Kirchoff's Law.
$\mathbf{B}_\nu(T)\Delta\nu$ = energy emitted cm^{-2} sec^{-1} in the frequency range ν to $\nu + \Delta\nu$ by a black body at temperature T.
$\mathbf{B}(T)$ = $\int_0^\infty \mathbf{B}_\nu(T)d\nu$ = total energy emitted by a black body = σT^4 by Stefan's Law.
$\mathbf{E}(T,m)$ = $\int_0^\infty E_\nu(m)\mathbf{B}_\nu(T)d\nu$ = total energy emitted by a non-black body.
$e(T,m)$ = $\mathbf{E}(T,m)/\mathbf{B}(T)$ = total emissivity.
$A_r(m)$ = $\int_r A_\nu(m)d\nu$ = integrated absorption, where the integral embraces the rth absorption band.
b_ν = brightness.
s_ν = brightness per unit length of an air column.
n = refractive index.

2. Solar radiation

In this section the amount and nature of the incident and reflected radiation will be discussed, more detailed comment on the absorption and scattering processes being postponed.

2.1. *The sun as a source*

In the visible region the distribution of energy in the solar spectrum and the total energy of the radiation at the confines of the atmosphere are approximately those of a black body at a temperature of about 6000°K. Departures from the black-body distribution are of great interest to solar physics, and may have some importance as far as direct thermal effects in the lowest 20 km of the atmosphere are concerned. For example it seems probable that certain solar disturbances are accompanied by the emission for short periods of a great excess of radiation in the extreme ultra-violet, while spectrograms obtained recently by high-altitude rocket show that in the near ultra-violet the energy is normally less than that appropriate to a 6000°K black body. Adel has obtained solar spectrograms suggesting a higher colour temperature than 6000°K in the infra-red, and recent measurements of the steady intensities in the centimetre-wave region demand effective temperatures of order 10^6 °K.

The earth's closest approach to the sun is at the northern winter solstice, the incident radiation being at this time about 7 per cent. greater than at its minimum. The mean value of the solar constant, obtained by the observers of the Smithsonian Institution, is about 1·94 cal cm^{-2} min^{-1}. (On the 1913 Smithsonian scale of radiation. Both this and the Ångström scale are in common meteorological use. For a discussion of the relation of these scales to each other and to an absolute standard see Nicolet (1948)). The observation of the solar constant is a matter of some difficulty and involves very substantial extrapolation ; on this account the claim of the Smithsonian observers to have found day-to-day variations of magnitude one or two per cent. is not universally accepted, and measurements by different methods are obviously desirable. The possibility of variation of the solar constant on a geological time scale is generally conceded, and is a prominent feature of the " accretion " theory of solar behaviour recently developed by Hoyle, Lyttleton, and their colleagues. (Hoyle 1949).

2.2. *The distribution of solar energy at the limit of the atmosphere*

The solar energy incident on unit area parallel to the earth's surface at the limit of the atmosphere was first computed by Angot. Some examples are given in Table I. The peculiarities of the distribution are clear from this abridged table,

TABLE I. Inward flux of solar radiation at the limits of the atmosphere, in cal cm^{-2} day^{-1}.

	21 March	21 June	23 Sept.	21 Dec.
80°N.	155	1060	153	0
60°N.	447	979	442	50
40°N.	686	991	677	318
20°N.	841	935	832	605
0°	896	790	885	843
20°S.	841	567	832	999
40°S.	686	297	677	1060
60°S.	447	47	442	1046
80°S.	155	0	153	1132

particularly the sharp summer peak at the poles, the very even distribution at the solstice over the summer hemisphere, and the existence, for almost three parts of the year, of a strong gradient in the latitudes about 50°.

2.3. *The distribution of solar energy over the earth's surface*

The distribution at the earth's surface can only be firmly established by an extended programme of measurement which is yet in its earliest stages. It is clear that the vertical flux of solar radiation (direct and scattered) is one of the most important climatic elements, but difficulties of maintenance and standardisation of the necessary instruments have precluded extensive measurement and resulted in one of the most serious gaps in meteorological knowledge. Apart from much more detail in the U.S.A. little can yet be added to tables such as that given by Süring (1938) from which the examples of Table II are extracted.

TABLE II. Examples of the mean total inward flux of solar radiation measured at the surface in cal cm^{-2} day^{-1}.

		March	June	Sept.	Dec.
Abisko	68·4°N.	135	455	141	(-)
Washington	39 °N.	348	525	375	168
Batavia	6 °S.	390	365	463	355

2.4. *The albedo of the earth*

A total of about 40 per cent. of the solar radiation is scattered back to space without change of wavelength. This is clearly a major factor in the heat balance of the earth, and its investigation, both for the planet as a whole and for various types of surface, is of great importance. Practical investigation of the albedo of the planet requires some type of extra-terrestrial observation and is typified by the work of Danjon (1936) who made a long series of visual photometric comparisons of the sunlit and earthlit segments of the moon, in white light and through red and blue filters. Danjon's mean value for the visual albedo was 39 per cent., greater for blue light and less for red. About half the incident solar energy lies in the infra-red, and the total albedo is required by the meteorologist. Fritz (1949) has recently attempted to extrapolate Danjon's work to obtain a total albedo for the planet. He considers separately the scattering at the surface, by clouds, and by air, and concludes that the total albedo is about 35 per cent., a value rather lower than that hitherto accepted. Confirmation of this result is urgently needed. It might be obtained by extensive observation of the visual and total albedos of various surfaces, of which cloud is by far the most important. A few early observations of the total albedo of cloud by Aldrich (1919) gave a value of 78 per cent., sensibly independent of thickness, and this was accepted by Simpson and others in studies of the heat balance of the earth. More recent observations from aircraft by Neiburger (1949) and Fritz (1950) of the total albedo of various clouds have suggested considerable variation with type and thickness, in agreement with the early work of Luckeish (1919) on the visual

bedo, but a preliminary note (*Harvard Report*, 1949) suggests that Hand has led to confirm these measurements, and indeed has obtained the same result Aldrich. The mean suggested by Fritz for all types of cloud is 50 per cent. to per cent., which is so different from Aldrich's value that we must hope for an rly resolution of the discrepancy. Theoretical considerations of scattering of oud particles have been used by Neiburger and Fritz in support of their proposals.

Fritz suggests that of the 35 per cent. of radiation returned to space only 2 to per cent. has been scattered from the earth's surface, so that reflection there is latively unimportant in the heat balance of the whole planet. It is, however, a ajor factor in the local heat balance, and many measurements have been made. ost of these are of the visual albedo which, for practically all land surfaces is less an the total albedo. Grassland, for example, with a total albedo of about 20 per nt., reflects only 5 to 10 per cent. in the visual. (The high reflectivity of green getation in the photographic infra-red is well known). Water and snow or ice rm the most extensive natural surfaces ; their albedo varies considerably with solar evation, but overall mean values for the total albedo of 9 per cent. and 60 to 70 r cent. respectively seem appropriate. Kuiper (1949) has studied the albedo of ow and finds that whilst it is almost a perfect diffuser in the visible it behaves in e infra-red as though the radiation had passed through a water cell of thickness mm to 20 mm, according to the surface condition. A 20 mm water cell is quite ack beyond 1·5 μ.

Measurement of total albedo involves the same instrumental problems as measure- ent of incoming solar radiation, and is obviously of equal importance. It is parti- ularly desirable that aircraft observations similar to those of Fritz should be made tensively in all types of surface and cloud conditions.

Finally we may draw attention to the relatively large variations, up to 5 per nt., found by Danjon in the visual albedo, and considered by him to be real and ue to variations of cloud amount in the segment of the earth reflecting during his bservations. This effect, if confirmed, might well be an important factor in the ersistence of weather types.

3. Terrestrial radiation

In this section the radiation originating at the earth and in the atmosphere will e discussed, details of the nature of the radiation, and of the physical processes nvolved in its emission, being postponed to section 7.

3.1. *The equation of radiative transfer*

In Fig. 2, A is a point in a uniformly stratified atmosphere. We wish to deter- ine the flux of radiation through A, and consider in the first place a single radiating onstituent of density $\rho(z)$ at a distance z from A in the vertical. It is convenient o use the quantity of radiating material m as a variable. The radiation at A is not sotropic, *e.g.* in Fig. 2 the radiation originating in element dz passes through an bsorbing layer of variable thickness $m\sec\theta$. It has however been shown that the

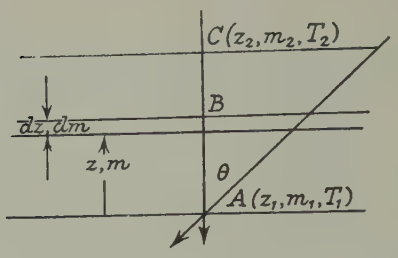

Figure 2. Radiating slab.

flux of radiation from an isothermal slab of radiating material can be related to the intensity of radiation in any direction (conveniently the vertical) by a numerical factor, and there is no loss of generality in considering vertical radiation only. If $E_\nu(m)$ is the emissivity of the isothermal slab of thickness m for radiation of frequency ν the flux at A of radiation of this frequency originating in the element dm is, by definition

$$\mathbf{B}_\nu\{E_\nu(m+dm)-E_\nu(m)\} = \mathbf{B}_\nu \partial/\partial m\{E_\nu(m)\}dm$$

and the total flux, for a slab of thickness M for all frequencies

$$\mathbf{E}(,T,M) = \int_0^\infty \int_0^M \mathbf{B}_\nu E_\nu'(m)dmd\nu$$

(Note that we write E' for $\partial E/\partial m$, not $\partial E/\partial z$). This is the equation of radiative transfer in an isothermal atmosphere. It has been tacitly assumed that $E_\nu(m)$ is a function only of the amount of radiating matter, not of its state (*e.g.* pressure and temperature). With these assumptions the equation can be established for an atmosphere in which pressure and temperature vary. Problems of radiative transfer in the atmosphere have in the main been attacked by integration of this equation, using various devices, justified more or less convincingly on empirical grounds, to adjust the results to compensate for the undoubted variation of $E_\nu(m)$ with pressure and temperature.

The problem of integration over ν reduces to the determination of the total emissivity $e(T,m)$ for all values of m. The complexity of the emissivity-frequency relation precludes any rigorous approach, and the various methods which have been used will be discussed later.

The integration over m, *i.e.* determination of

$$\mathbf{E}(T,M) = \int_0^M \mathbf{B}(T)e'(T,m)dm$$

remains. $\mathbf{B}(T)e'(T,m)$ is a function of T, and in each atmospheric problem there is an observable relation between m and T which may take a wide variety of forms not expressible analytically. A graphical or numerical method of integration is thus a physical necessity. An obvious possibility is a chart with axes $\mathbf{B}(T)e'(T,m)$ and m, with isopleths of T, on which the observed values of T and m may be plotted and the required flux evaluated as an area. Such a diagram is called a radiation chart.

3.2. *Radiation charts*

Several of these devices have been proposed ; three will be mentioned here in illustration. The unmodified form of the transfer equation is not satisfactory in practice owing to the nature of the variation of $e'(m)$ at small m. Mügge and Möller, the pioneers of this method, illustrated a chart using the unmodified equation, but did not propose it for practical use. It is shown diagramatically in Fig. 3a, the flux of radiation at A (Fig. 2) due to the layer AC being given by the shaded area. Mügge and Möller obtained a curve of $e'(m)$ against m by graphical integration of a crude generalisation of the water-vapour spectrum, for various values of m.

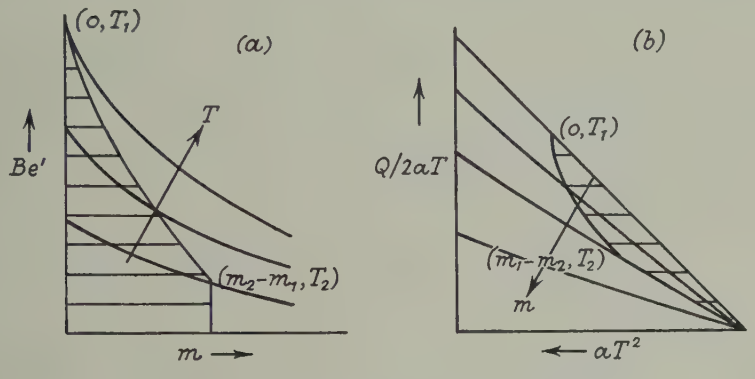

Figure 3. Illustrating types of radiation chart
(a) Original Mügge - Möller chart
(b) Elsasser chart

Radiation charts of practical value are based on transformations of the transfer equation. Perhaps the best known is that of Elsasser (1942), which uses the form

$$\mathbf{E} = \Big[\mathbf{B}(T)e(T,m)\Big]_{m_1}^{m_2} - \int_{m_1}^{m_2} \mathbf{B}'(T)e(T,m)dm$$

$$= \Big[\mathbf{B}(T)e(T,m)\Big]_{m_1}^{m_2} - \int_{T_1}^{T_2} \frac{\partial}{\partial T}(\mathbf{B}T)e(T,m)dT$$

Writing $Q = \partial\mathbf{B}/\partial T(1 - e)$, the abscissae of Elsasser's chart are aT^2, the ordinates $Q/2aT$, a being chosen to give a suitable scale. The chart has the appearance shown in Fig. 3b, the shaded area showing the flux at A of radiation originating in the slab AC. Elsasser's determination of e was developed in much more detail than were those of previous workers.

Possibly the simplest form of radiation chart is that typified by the Kew chart, described by Robinson (1950). This is based on the form

$$\mathbf{E} = \int_{m_1}^{m_2} \mathbf{B}(T)de(m) = \Big[\mathbf{B}(T)e(m)\Big]_{m_1}^{m_2} - \int_{T_1}^{T_2} e(m)d\mathbf{B}(T)$$

and the assumption that $e(m)$ is independent of temperature. The relation between $e(m)$ and m incorporated in this chart is purely empirical. For the smaller values

of m the direct experimental results of Elsasser and F. A. Brooks (1941) are used; for large m, observations of atmospheric radiation from various altitudes, reduced to isothermal conditions by a method proposed by F. A. Brooks, are adopted. This reduction can be justified only if $e(m)$ is independent of T, an assumption which involves neglect of the changes in $\mathbf{B}_\nu$ with temperature as well as the previous assumption of temperature independence of E_ν.

3.3. *Computation of the divergence of radiative flux*

The methods so far described are designed to allow calculation of the radiative flux at any level in the atmosphere. The meteorologist is mainly concerned with the temperature change due to radiation, *i.e.* with the divergence of the flux. Existing radiation charts may be used to give flux differences over finite layers, but if the method is used for thin layers it is found that the areas on the chart are small and of awkward shape. A flux-divergence diagram would reduce the work involved and allow determination of the rate of temperature change in a very thin layer. Such a diagram could be based on the equation

$$\frac{\partial T}{\partial t} = -\frac{1}{\Sigma \rho c_p}\frac{\partial \mathbf{E}}{\partial z} = -\frac{\rho}{\Sigma \rho c_p}\mathbf{E}' = -\frac{\rho}{\Sigma \rho c_p}\int\int \mathbf{B}_\nu E_\nu''(m)dmd\nu,$$

where $\Sigma\rho$ is the air density, or on a transformation. No diagram of this type appears to have been published, but the possibilities have been discussed by Cowling (1950b) who points out some of the difficulties of this apparently attractive solution. Two numerical methods may be mentioned. The first, used by D. L. Brooks (1950) employs the particularly simple form of the flux-divergence equation

$$\frac{\Sigma \rho c_p}{\rho}\frac{\partial T}{\partial t} = -\left[\mathbf{B}(T)e'(m)\right]_{m_1}^{m_2} + \int_{T_1}^{T_2} e'(m)d\mathbf{B}(T)$$

which results from the transformation used in the Kew chart. The second, due to Bruinenberg (1946) has been set out in considerable detail in a paper comparable in importance with that of Elsasser, but perhaps less widely known. Developed independently, it employs Elsasser's form of the transfer equation. Differentiating this we have

$$\begin{aligned}\frac{\Sigma \rho c_p}{\rho}\frac{\partial T}{\partial t} &= -\left[\mathbf{B}(T)e'(m)\right]_{m_1}^{m_2} + \int_{T_1}^{T_2} Q'dT \\ &= -\left[\mathbf{B}(T)e'(m)\right]_{m_1}^{m_2} + \int_{m_1}^{m_2} Q'\{\gamma(z)/\rho(z)\}dm\end{aligned}$$

where $\gamma(z)$ is the local lapse rate. Bruinenberg's notation is rather different. He employs a function $S(m,T)$ such that $S' = -\mathbf{B}e'$ and $\frac{\partial S}{\partial T} = Q$. His method lends itself to numerical rather than graphical treatment, and he gives extensive tabulations of the functions S' and $\partial S'/\partial T$ both for water vapour and for carbon dioxide. The emissivities on which these are based are derived from the work of Möller and Schnaidt (1939), the treatment being less detailed than that of Elsasser but the final result being, as pointed out by D. L. Brooks (1950), very little different.

Computation of flux divergence has so many apparent advantages over more widely used methods that it is as well to point out that some of them may be illusory. The inaccuracy of computation of temperature change in a thin layer is obvious when using a flux diagram ; when the divergence method is used the same inaccuracy is hidden in the tables of e'. Differentiation of the empirical or semi-empirical e — m relation is the crux of the problem, and cannot be avoided.

3.4. *Limitations of existing methods of computing radiative flux*

It will be clear that there is a good deal of empiricism and simplification in even the most complicated of existing radiation diagrams. It follows that their applicability to any problem should be critically examined before use and we will consider in turn some causes of their inadequacy.

The one to which most attention has been paid is our imperfect knowledge of the details of the water-vapour spectrum. Up to about 1945 radiation diagrams were continually being amended as more information became available on the positions, intensities, and widths of the lines in the rotation band ($\lambda > 15\ \mu$). The 6 μ band has recently been analysed in similar detail, and a more reliable value of the line width (see section 7) has been obtained, so that in this respect the meteorologist's application now lags behind the physicist's provision of data.

Other limitations are more fundamental. The most obvious is that the charts described apply to a single radiator, and that if two radiators are active in the same frequency range the method cannot be adapted rigorously to deal with them. Fortunately only two radiators, water and CO_2, are likely to be important in the lower troposphere. The relevant part of the CO_2 spectrum, a band centred about 15 μ, is of very great intensity, both absolutely and relative to the water emission in the same region. In the radiation charts described CO_2 has therefore been treated as a black body in this limited region (which contains almost 20 per cent. of full radiation at the temperatures concerned), an approximation which appears to be satisfactory throughout the lower atmosphere except in the immediate vicinity of the surface, where temperature gradients are liable to be very large. A closer approximation may be made by constructing a carbon-dioxide-radiation chart on the same lines as that for water, and Bruinenberg has published the appropriate tables for his method. Ozone has a negligible effect on flux divergence near the surface, but its importance increases with height. It is ignored in all the methods we have discussed. The remaining important radiator, particulate matter, can be accounted for when it takes the form of a complete layer of cloud, in which case each surface may be regarded as a black body at the appropriate temperature. Broken cloud, dust, *etc.*, can only be treated by *ad hoc* methods of very doubtful accuracy.

It will be recalled that the transfer equation on which the charts are based is valid only if $E_\nu(m)$ is independent of pressure and temperature. The nature of the spectrum is profoundly affected by these variables, and attempts have therefore been made to " correct " the charts for changes of p and T. The only feasible method of correction is adjustment of the value of m, and in both the Elsasser and Kew charts m is replaced, following consideration of a variety of laboratory evidence, by $m(p/p_0)^{\frac{1}{2}}$.

Temperature effects have been even less-satisfactorily treated ; here we need only note that none of the methods we have mentioned incorporates any attempt to correct for the temperature variation of $E_\nu(m)$, and the Kew chart even ignores the temperature variation of $e(m)$.

It may be concluded that, since the methods we have described have been tested at the surface, they may be used with reasonable confidence there and in the lowest kilometre or two of the atmosphere. At greater heights their indications must be considered unreliable, and in the region of the tropopause possibly valueless. There is, however, little doubt that their reliable extension to these heights is one of the most urgent meteorological problems. It is certainly not the least difficult.

4. Thermal effects of solar and terrestrial radiation

Radiation causes temperature changes in the atmosphere both directly by absorption in the air and clouds and indirectly by absorption at the ground and consequent convective transfer of sensible and latent heat. The total energy available in any locality is given by the difference of inward and outward flux at the limits of the atmosphere, whereas the direct heating at any point is given by the divergence of total radiative flux. The meteorologist requires a complete description of the thermal effects at all points in the atmosphere. As is clear from the previous sections the present position of this study is far from satisfactory. It will be summarised in this section, but we are anxious that the figures to be quoted will not be separated from the warnings concerning their reliability.

4.1. *Divergence of the flux of terrestrial radiation*

Many examples have been published of computations of the heating and cooling effects of terrestrial radiation, using radiation charts or numerical methods. The main feature of the results is that, except close to a heated land surface, there is a general cooling. In the absence of cloud, considering only water vapour, and taking average conditions, a fairly uniform cooling of between 1 and 2°C day^{-1} from the surface to within the stratosphere is indicated. Below a cloud layer the cooling is considerably reduced ; at and close to its upper surface the rate of cooling is greatly increased. The effect of the observed distribution of cloud on the average computed cooling rates is to reduce the cooling in the lower levels to from 0 to 1°C day^{-1} according to latitude, increase it to 2 to 3°C day^{-1} in the middle troposphere, and make little change in the region of the tropopause.

It is clear from our discussion of the limitations of the methods of computation that the accuracy of these estimates of cooling in the upper atmosphere is very doubtful, and measurements of terrestrial radiation, and particularly its divergence, at high altitudes are urgently required. Goody (1950) has discussed the possibility of such measurements. The instrumental difficulties are formidable, in particular the need for a sensitive, rapid, and stable detector of radiation with black- or grey-body characteristics over a wide range of frequency.

4.2. *Absorption of solar radiation in the atmosphere*

The atmospheric constituents responsible for the main exchange of terrestrial diation are also important as absorbers of solar energy. The Hartley-band absorp-n of ozone is practically complete at heights above those in which we are interested, t the Chappuis absorption is active in the lower stratosphere. Direct absorption ozone and carbon dioxide is probably negligible in the troposphere compared th that of water vapour and cloud.

Direct absorption in the lower stratosphere has been discussed by Karandikar 946) ; his results indicate heating of the order of 0·1°C day^{-1}. Absorption by ater vapour in the troposphere has been computed by many authors — Tanck 940) gives a detailed treatment with examples of the mean heating between the rface and 5 km at Hamburg for different weather types throughout the year. is values range from 0·1°C day^{-1} in midwinter to 0·6°C day^{-1} in midsummer. bsorption by cloud has been less thoroughly examined but Hewson (1943) and eiburger (1949) have suggested theoretical treatments which indicate that clouds out 1 km thick may well absorb between 5 and 10 per cent. of the incident solar nergy.

The observation of the divergence of flux of solar radiation is a much simpler roblem than the corresponding one for terrestrial radiation. Such observations ave been reported by Neiburger (1949) in cloud and Fritz (1948) in clear air. he former obtained rather scattered figures up to about 7 per cent. for clouds of ickness up to 2,000 ft, in satisfactory agreement with his theoretical estimates. he latter made measurements of the net flux of solar radiation in cloudless con-itions which indicated a heating of 0·06°C hr^{-1} between the surface and 7,000 ft nd 0·04°C hr^{-1} between 7,000 ft and 10,000 ft, again in very satisfactory agreement ith the predictions of Tanck and others. Many more such measurements, and articularly their extension to the lower stratosphere, are desirable.

4.3. *Sources and sinks of heat in the atmosphere*

Combination of studies of divergence of radiative flux with those of total adiation balance locates the sources and sinks of heat in the atmosphere. The dis-ribution and strength of these gives the pattern of the hydrodynamic heat flux. impson (1929) was the first to attempt a world-wide survey of the field of total adiation. The physical basis of his computations cannot now be accepted (although e was the first to consider the atmosphere as other than a grey body), but a recent ess-detailed computation by Raethjen (1950), using modern methods, has yielded emarkably similar results. Both these authors were concerned with mean values ver broad zones of latitude and periods of a month or more. Table III and Fig. 4 re based on Raethjen's results. The general pattern is clear ; the surface is a ource everywhere except in the winter polar regions and adjacent continental areas, he troposphere everywhere a sink, especially at the level of the uppermost cloud urface. The radiation field is such as to give rise to a mean hydrodynamic flux f heat upwards and from the sub-solar region towards each pole throughout the ear. This is an eminently reasonable picture, and is usually accepted without uestion. It is therefore most important to remember the limitations in its deri-

TABLE III. Radiation balance of the zones of latitude. Mean values over the winter and summer half-years in cal cm^{-2} min^{-1} (based on a table by Raethjen).

	Winter				Summer			
Zone	90°–60°	60°–40°	40°–20°	20°–0°	0°–20°	20°–40°	40°–60°	60°–90°
Fraction of earth's surface	0·07	0·11	0·15	0·17	0·17	0·15	0·11	0·07
Net short-wave radiation	·023	·118	·258	·353	·414	·425	·349	·243
Net long-wave radiation	−·292	−·292	−·312	−·300	−·296	−·315	−·304	−·308
Total radiation balance	−·269	−·175	−·053	+·053	+·118	+·110	+·045	−·065
Radiation balance of tropospheric air	−·212	−·167	−·137	−·111	−·087	−·099	−·091	−·125
Radiation balance at surface	−·057	−·008	+·084	+·163	+·205	+·209	+·136	+·061

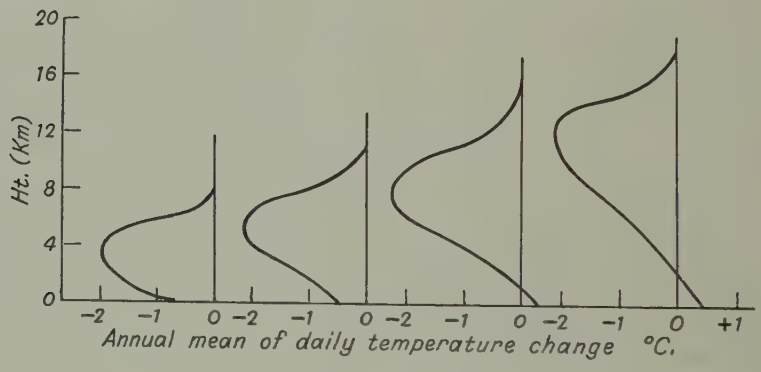

Figure 4. Radiative temperature changes in the troposphere for various zones of latitude. The curves, from left to right, refer to latitude zones 60°-90°, 40°-60°, 20°-40°, and 0°-20° respectively (after Raethjen).

vation. As we have seen, direct observation plays a negligible part, and the results depend considerably on three factors each open to much doubt ; the amount of cloud and the height of its upper surface, the albedo of cloud, and the validity of radiation charts and similar devices at low temperatures and pressures. According to Table III, the net radiative income is only about 15 to 30 per cent. of the computed inward or outward radiation in each zone of latitude, *i.e.* it is comparable in magnitude with quite likely errors in its components. Table III may therefore be misleading in important details ; as a particular example it may be said that there is no conclusive evidence from radiation studies of radiative excess in the equatorial regions and the consequent existence of a mean flow of heat across the sub-tropical high-pressure belts.

One important aspect of this study which has been almost entirely neglected is the difference in the radiation field of different weather systems. There may well be equally significant differences between the radiation régimes of a depression and a cloudless anticyclone in the same latitude as between the mean régimes of widely different latitudes, and this must have considerable bearing on the stability of these systems.

4.4. *Radiation and the diurnal temperature variation*

This is the appropriate place to mention briefly the aspect of radiation most familiar to the weather forecaster — the diurnal temperature change, and in particular the "radiation night" and "radiation fog." The primary variable is the radiative flux at the surface ; others to be considered are heat conduction in the soil and the convection of sensible and latent heat. Many authors have discussed the problem, some ignoring conduction in the soil and making assumptions concerning the nature of convection, some ignoring convection. It may safely be said that no satisfactory treatment reasonably free from empiricism has been produced. Examples of recent treatments are Groen (1947), Fleagle (1950) and Knighting (1950). Rider and Robinson (1951) have emphasised the importance of radiative transfer within the lowest layers of air. Using radiation-chart methods they show that the flux divergence close to the ground is frequently some 10 to 100 times that required to give the observed temperature changes, an effect which must be compensated by convection. In very calm conditions they find temperature distributions appropriate to a state of radiative quasi-equilibrium. It appears from this that radiation exerts a controlling influence on the very large temperature gradients often observed near the ground.

5. Infra-red spectroscopy and atmospheric problems

5.1. *The importance of infra-red spectroscopy*

Infra-red spectroscopy has had a continuous association with problems of atmospheric physics since the discovery of the infra-red by Herschel in 1800 when working on the solar spectrum. It can be seen from Fig. 1 that most of the strong atmospheric absorptions lie at wavelengths greater than 0·8 μ. Recent advances in our knowledge of atmospheric absorption and attenuation have therefore been largely bound up with advances in the technique of infra-red spectroscopy. These have been very rapid in recent years owing to the development of synthetic crystals for prisms, to the application of electronic methods, and mainly to the development of sensitive and fast radiation detectors, such as the Hilger-Schwarz thermocouple, the Golay cell, and lead-sulphide and -telluride photoconductive layers. The most recent results of the application of these methods to atmospheric problems will be outlined in the subsequent paragraphs.

It is convenient to discuss separately grating spectra and prismatic spectra. With the best grating equipment it is now possible to achieve a resolution of 0·1 cm^{-1} (0·001 μ at 10 μ), while with prismatic spectra the limit is more nearly 5 cm^{-1}. This is a difference of kind rather than degree, since most infra-red absorption bands consist of lines about 1 cm^{-1} apart ; prismatic spectra can therefore only show the broad structure of a band while a grating spectrum will usually resolve the detailed fine structure.

5.2. *Prismatic solar spectra*

Fowle (1917) succeeded in recording the solar spectrum from the visible to 13 μ, and at that time the main features of the spectrum were known and, in the main, rightly attributed to absorptions by oxygen, water vapour, carbon dioxide

and ozone. Recent work has not extended our knowledge of the major absorptions, but has mainly been concerned with minor atmospheric constituents. Adel has been largely responsible for redirecting attention to the solar spectrum, and his work is described in a series of papers in the *Astrophysical Journal* (see Adel 1949a). He not only had the advantage of better infra-red techniques, but also of the dry climate of Flagstaff, Arizona, and many of his most interesting results are connected with the greatly reduced absorptions in the 6·3 μ and the rotation bands of water vapour.

A strong doublet band centred at 7·8 μ, which shows up clearly if the absorbing path of water vapour is less than 1 cm of precipitable water, was recognised as one of the fundamentals of nitrous oxide. This discovery aroused some interest since nitrous oxide is a gas which might possibly be formed by photochemical reactions, and like ozone, be concentrated in the stratosphere, therefore exerting considerable influence upon the outgoing stream of terrestrial radiation. A search for further bands was made by Shaw, Sutherland and Wormell (1948) and two were detected at 3·9 μ and 4·5 μ whilst other experimenters have subsequently detected more bands at shorter wavelengths (Goldberg 1950). It appears therefore that nitrous oxide is of world-wide occurence, and it is usually stated that the total quantity in the atmosphere corresponds to 8 mm at S.T.P., although Adel's spectra indicate little more than a third of this amount above Flagstaff. (See also sections 5.3 and 5.6).

Another achievement of Adel (1942) has been the detection of a window near 19 μ between the water-vapour rotation band and the 15 μ carbon-dioxide band. This has extended the solar spectrum to 24 μ and although it mainly brings to light known groups of water-vapour lines, it has also shown that there are no other strong absorptions in this region which might be of importance at great heights where water vapour absorbs less strongly.

The amount of water vapour in the atmosphere corresponds to one or two miles of saturated air under ground-level conditions, so that observations on days with differing humidity and with various solar zenith angles can provide transmission data for water vapour which are practically unobtainable by other methods. In this way Adel and Lampland (1940a and b) have determined the transmissivity of water vapour over the range 5·5 μ to 14 μ, a range of great importance for the evaluation of the radiation flux at ground level. Unfortunately owing to the effect of pressure upon absorption it is not certain to which pressure level these results apply, although there is no doubt that the pressure does not differ greatly from one atmosphere.

5.3. *Grating solar spectra*

Grating spectra with a resolution of 1 cm^{-1} can resolve the fine structure of many atmospheric bands, and hence for purposes of identification it is possible to look for absorption coincidences not merely at the band centre but also at 100 or more line centres. Prismatic spectra often leave an element of doubt as to the correct assignation of an absorption band, but this doubt can invariably be removed by examination of the resolved line spectrum. In this way Adel (1941a) confirmed his identifications of the 7·1 μ HDO band and the 7·8 μ nitrous-oxide band using

grating spectrometer and a thermocouple as radiation detector. Adel (1941b) so mapped the spectrum from 2 μ to 14 μ with a grating spectrometer, revealing large number of lines, partly of solar and partly of telluric origin.

Groups of workers at Michigan, John Hopkins, Ohio State, and Liège Universi-es are engaged on the exploration of the infra-red solar spectrum, using grating pectrometers and, over the range 1 μ to 5·5 μ, photoconductive cells which are far nore sensitive than thermocouples in that range. Many lines due to minor con-ituents of the atmosphere have been investigated ; we may mention especially Migeotte's (1948) identification of methane. This seems to be of world-wide ccurrence, in amount corresponding to a layer 11 mm thick at S.T.P., with a mean emperature of emission of — 33°C.

The possibility of mean-temperature determinations is one of the most valuable dvantages of high-resolution work. Quantum theory can be used to predict the elative intensities of lines at any temperature, and therefore measurement of these ntensities leads to a temperature which may be referred to as the emission tempera-ure. Such a thermometer can give results to within $\pm$ 1°C or so, but unfortunately, ince matter is distributed in the atmosphere under a temperature gradient, this emperature has no very precise meaning.

5.4. *Long absorbing paths at ground level*

Instead of using the whole atmosphere as an absorption tube it is clearly useful o perform more controlled experiments at ground level with an artificial source nd a long absorption path, either open, in which case the atmosphere at ground evel is sampled, or closed. In either case the path must usually be very long in rder that fairly large quantities of gas may be investigated at a dilution with air omparable to that occuring in the atmosphere. It is known that a number of closed bsorption tubes, in which long paths are obtainable by multiple reflections, have een built in America (see for example Herzberg 1949) and that work is in progress pon the absorptive properties of atmospheric gases which will be of great importance o thermal studies of the atmosphere. As yet however no details have been published, nd this is probably indicative of the technical difficulties which can arise.

Open-path measurements have been performed in this country with a path ength of 2 km over the sea (T.R.E./A.R.L. 1949). Absolute measurements were nade using an ingenious comparison with absolute transmission measurements n the visible spectrum *i.e.* visual-range measurements, and reliable scattering and bsorption data were obtained from the recordings. The spectroscope used was rismatic and the resolution not high. Apart from absorptions by water vapour nd carbon dioxide, a few absorptions caused by minor constituents were recorded ncluding the HDO fundamental at 3·67 μ and nitrous-oxide bands at 4·5 μ and ·8 μ. The speculations which have been made about the presence of nitrous xide in the stratosphere would not lead one to expect finite quantities at ground evel ; these experiments indicate therefore that nitrous oxide, like carbon dioxide nd helium, is probably mixed in equal proportions with air at all levels in the tmosphere.

5.5. *Solar spectroscopy at high altitudes*

Since solar spectroscopy leads only to information about the atmosphere above the level of observation, it is desirable to be able to record solar spectra at all altitudes, as has been done with ultra-violet spectra from rockets and balloons. Mountain observatories can be used for altitudes up to 4 km, but this does not lead to results of great fundamental interest. Since infra-red spectroscopes are heavy, delicate, and slow in operation, attempts have so far only been made to record spectra from aircraft.

A group at Johns Hopkins under J. Strong (1949a) has succeeded in installing spectroscopic equipment in a B29 aircraft, and spectra have been recorded at four levels up to 11 km. The resolution was poor, but recordings made from 2 μ to 24 μ showed all the features incorporated in Fig. 1. The 2·7 μ and 6·3 μ water-vapour bands were weak at 11 km. The 4·3 μ carbon-dioxide band does not fall off as rapidly as the water-vapour bands and is still strong at 11 km. The 9·6 μ ozone band is as intense at the maximum altitude as at ground level while the 15 μ carbon-dioxide band, although greatly diminished at high altitude, is still completely absorbed at the band centre. A weak band near 7·8 μ was originally attributed to nitrogen pentoxide, but it is more likely to be the 7·8 μ nitrous-oxide band, and its weakness confirms the conclusion reached in the last paragraph that nitrous oxide is not concentrated in the stratosphere. No quantitative discussion of these spectra has yet been made, although it is of the greatest importance to do so.

Goody (1950a) has also reported some preliminary measurements from a Mosquito aircraft flying at 9 km. The aim of these experiments was the quantitative estimation of water vapour, so only the 6·3 μ band of this gas was recorded. The precipitable water above 9 km for three days in Feb. 1949 was found to be about 5.10^{-3} cm, and rough estimates could be made of the water-vapour content of the stratosphere. By taking spectra at two levels on the same day it was possible to measure water-vapour concentrations and to obtain some check upon the accuracy of the frost-point hygrometer at low humidities. Agreement was found between the two methods within a fairly large permissible error.

5.6. *Emission spectroscopy*

In a spectroscope with a thermal detector such as a thermocouple, the observer is measuring the exchange of radiation between a body at room temperature and whatever hot body is imaged upon the slits. Modern detectors are now so sensitive that it is possible to work with only a very few degrees difference in temperature between the " hot body " and the thermocouple. Under these conditions however the spectral resolution will be low because it is necessary to use very wide slits. If the thermocouple is assumed to have the same temperature as the earth's surface, and if the long-wave radiation from the atmosphere is allowed to fall upon the spectrometer slits then the spectral distribution of the radiation exchanged between earth and atmosphere may be measured. This development is of the greatest importance for the construction of radiation charts although results so far have not been sufficiently precise to be of general use.

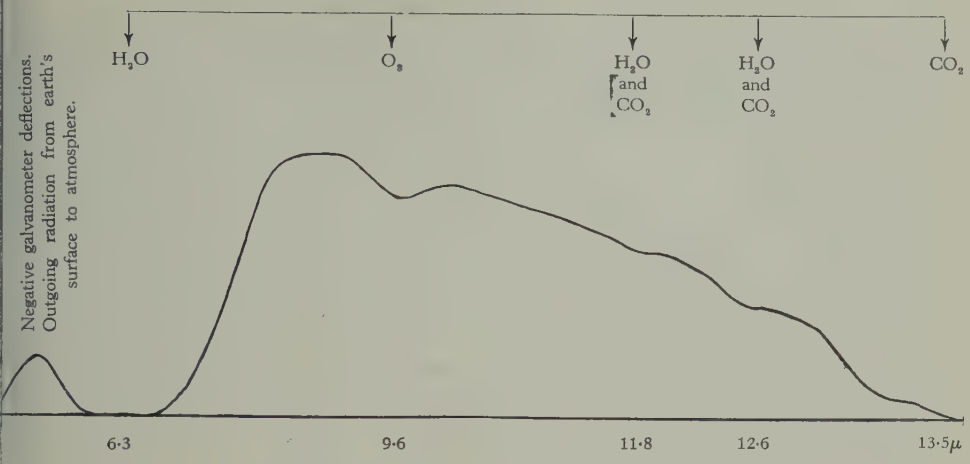

Figure 5. The radiation exchanged between the earth and the atmosphere (Adel 1950).

Fig. 5 shows a spectrum recorded by Adel. If the atmosphere were transparent this would be the emission curve for the thermocouple which is approximately a 300°K black body. The outline of the spectrum in Fig. 5 resembles such a black-body curve except that there are deviations in the major atmospheric absorption bands. At 6·3 μ and again beyond 13·5 μ the atmosphere absorbs strongly, and therefore behaves like a black body at ground temperature, and no exchange of radiation takes place. The small ozone emission at 9·6 μ can be measured fairly accurately, and if a solar absorption spectrum is also available, it is possible to calculate the emission temperature of ozone by a simple application of Kirchoff's Law. By this means Adel (1950) found an emission temperature for ozone of — 40°C in accordance with our knowledge from other sources.

By a similar method Adel has also measured the emission temperature of nitrous oxide (which does not show up at 7·8 μ on Fig. 5), and finds it to be about + 5°C, confirming the indications previously mentioned that nitrous oxide exists largely in the lower troposphere. Strong (1949 b) has also measured ozone emission temperatures by a method essentially similar to that used by Adel. Although these measurements of emission temperature are very valuable, their precise significance is not clear, as has already been pointed out in section 5.3.

5.7. *Pressure measurements*

Infra-red absorption bands have the complicating property of being strongly dependent upon the pressure as well as the amount of gas. Instead of regarding this as a difficulty it may be used as a convenient method of measuring the pressure level of any absorbing gas. Measurements have to be made on two bands each with a different pressure effect, and since the quantity of absorbing material is known to be the same for each band, it is in principle possible to determine the

pressure needed to give the observed intensity ratio of the two bands. This method has been applied by Strong (1941b) to the pressure level of ozone emission. Ozone is particularly suitable for this purpose since the ultra-violet Hartley band is independent of pressure while the 9·6 μ band is strongly pressure-dependent. Strong found the ozone layer to be between 15 and 30 km, in agreement with information from other sources.

These emission-pressure measurements are subject to the same difficulty of interpretation as the temperature measurements, namely that the exact meaning of the pressure measured is not clear. Nevertheless Strong has demonstrated that it is possible to measure at least one valuable parameter concerned with ozone other than the total amount in the atmosphere. The whole ozone problem has so far proved so instructive and important that it seems a pity that this kind of measurement has not been further exploited.

6. The absorption and emission of radiation by atmospheric gases

6.1. *Introduction*

Beer's Law states that, for monochromatic radiation, transmission varies exponentially with the amount of absorbing material. Provided that calculations are performed, and measurements made, within a range of wavelengths which can be considered to be monochromatic, then this law is applicable under all circumstances met with in the lower atmosphere. The requirement of monochromatism is satisfied if the absorption coefficient varies only slightly over the range of wavelengths considered. This is normally the case for visible and ultra-violet but never holds for infra-red atmospheric absorptions. The best infra-red resolutions so far obtained are only comparable with the maximum line widths which exist, and therefore measured transmission, being the sum of many exponential functions, is not itself an exponential function. This is often referred to as the breakdown of Beer's law, although it is the experiment and not the law which is in the wrong, and it is with the difficulties which arise on this account that this section is concerned. The difficulties lie not so much in the fundamentals of infra-red absorption, as in the application to atmospheric problems. Inability to resolve line structure makes it difficult to determine such parameters as line width and line shape, and even if all such knowledge were available it is doubtful whether meteorologists would have the time to use it.

Absorption data may be collected with two alternative aims. A straightforward empirical approach is possible whereby averaged or smoothed absorptions are measured in the laboratory under a variety of conditions, in the hope that the results can somehow be applied to the atmospheric problem. The difficulty here is that it is impossible to imitate the distribution of matter under a pressure gradient, and therefore some extra knowledge is required to achieve the required aim. Alternatively it is possible to seek information about the fine structure of the absorption spectrum. This is the more fundamental approach, and since in principle it is possible to solve any atmospheric problem given the required information about fine structure, it will be given the greatest prominence in this section.

6.2. *The positions and intensities of absorption lines*

An infra-red molecular band is caused by the simultaneous change of both the rotational and vibrational quantum number. The vibrational transition determines the frequency of the band centre, and the numerous possible rotational transitions define the positions of rotation lines crowded around that band centre. It is possible to have a band where the vibrational quantum number has not changed, and in this case a pure rotation band appears, similar to other vibration-rotation bands except that the band centre is at zero frequency. A familiar example of this is the water-vapour rotation band, whose lines can be observed in the infra-red near 15 μ and also in the micro-wave region. Since any gas may have several bands, and each band may have very many lines, it is a very welcome achievement of quantum mechanics that, given a model of the molecule, it is possible to calculate the position of every line in the spectrum. In the cases of water vapour, carbon dioxide, and nitrous oxide, molecular models have been precisely established (see Herzberg 1944). There is still some confusion over the structure of ozone, but it will probably not be long before this too is well understood.

We require to know the shape and intensity as well as position of each individual line ; on neither of these subjects does quantum mechanics give exact information. It is possible to calculate transition probabilities and therefore line intensities from quantum mechanics, provided that additional information is available about the molecular model. In the case of the intensities in a pure rotation band, or the relative intensities in a vibration-rotation band, the problem is comparatively simple and the results of theoretical calculation should be reliable. This is valuable since the rotation band of water vapour is particularly difficult to observe, and in the case of vibration-rotation bands it is only necessary to determine a single parameter, *viz.*, the band intensity, or the sum of the individual line intensities. Yamamoto and Onishi (1949) have calculated the intensity of the water-vapour rotation band and hence have tried to calculate transmissions in this region of the spectrum. Although their calculation is open to criticism, it may be compared with similar work by Elsasser (1942) who normalised his results to agree to some extent with observation. There is substantial agreement between these authors, which suggests that in this case theoretical calculations may be significant. By the same token their disagreement on the 6·3 μ band of water vapour illustrates the well-known fact that calculations of band intensities for vibrational transitions can be very badly in error. Fortunately it is possible to measure this parameter.

The integrated absorption (see section 1 for definitions) is a relatively easy quantity to measure. If the quantity of absorbing material is very small the exponential can be represented by its first term, and

$$A_\tau = m \int_\tau k_\nu d\nu$$

This expression predicts that for small absorptions the integrated absorption varies linearly with amount of absorbing material, and the slope is simply the band intensity. Goody and Wormell (unpublished) working on the 7·8 μ and 8·6 μ bands of nitrous oxide have shown that it is quite possible to make this a fairly precise determination. The way is now open therefore to obtain all the information upon intensities which is likely to be required in atmospheric problems.

6.3. *Line shapes and widths*

There has been some confusion in the literature about the shapes of infra-red absorption lines, arising from the fact that direct observation is not yet possible. Few doubts can now remain however, since theoretical work, observation in the visible and microwave regions, and indirect evidence from the infra-red region, combine to give a consistent picture.

With slight simplification, the main factors influencing line broadening may be stated as follows (Margenau and Watson 1936, Weisskopf 1933, Plaskett 1947) :

(a) natural broadening caused by self damping,
(b) Doppler broadening caused by chaotic molecular motions,
(c) Lorentz broadening caused by collision damping,
(d) statistical broadening caused by the mutual interference of molecular force fields.

Taking the 7·8 μ nitrous-oxide band as a typical example, (a) gives a line width of $3{\cdot}4.10^{-4}$ cm^{-1}, (b) gives a line width of $2{\cdot}4.10^{-3}$ cm^{-1}, while (c) gives a line width of $0{\cdot}3\,p$ cm^{-1}, where p is the pressure in atmospheres. In the lower atmosphere therefore (a) and (b) may be dismissed from further consideration, although in the stratosphere Doppler broadening may well be the most important factor. It can also be shown from a simple theory of Bleaney and Loubser (1950), together with our knowledge of collision diameters, that statistical broadening is just not important in the atmosphere. It will probably be necessary to consider this problem in more detail when accurate measurements of the line widths for carbon dioxide and ozone are available. In the lower atmosphere therefore the Lorentz line shape characterised by the formula

$$k_\nu = k\alpha/\pi\{(\nu - \nu_0)^2 + \alpha^2\}$$

where ν_0 is the frequency at the line centre, is of primary importance.

Kinetic theory gives for the half line width the expression

$$\alpha = (4\pi)^{-1}\,\Sigma_i N_i(D + D_i)^2[2\pi kT(1/M + 1/M_i)]^{\frac{1}{2}}$$

N_i is proportional to the partial pressure, and in the case of near-infinite dilution in a neutral gas such as air, line width will be proportional to the total pressure. It is not possible to predict collision diameters ; they usually turn out to be larger than those given by viscosity measurements, and moreover they are not simply additive. It is therefore necessary to have at least one measurement of line width. In the case of collisions between polar molecules the line width may vary from line to line, although when one of the molecules involved is non-polar (like oxygen or nitrogen) this effect should not be very large. The final complication is that collision diameters are temperature-dependent.

The influence of this effect upon absorption is, however, small compared with those arising from the variations of pressure and amounts of absorbing gases in the atmosphere and it is also smaller than the effect of temperature upon line intensities, which can be calculated fairly precisely.

Measurements of line width in the infra-red spectrum are few. Adel (1947) has measured line widths in the water-vapour rotation band at S.T.P. and found,

$$2\alpha = 0{\cdot}24\ (\pm\ 0{\cdot}02)\ \text{cm}^{-1} \text{ at } 18{\cdot}64\ \mu,$$
$$2\alpha = 0{\cdot}22\ (\pm\ 0{\cdot}02)\ \text{cm}^{-1} \text{ at } 15{\cdot}99\ \mu.$$

the same band at 12,760 μ in the microwave region, Becker and Aulter (1946) nd, at a temperature of 318°K, that

$$2\alpha = 0{\cdot}174\ (\pm\ 0{\cdot}02)\ \text{cm}^{-1}.$$

e difference between these results is probably not significant.

6.4. *Absorption by a band spectrum*

A number of attempts have been made to calculate the transmission function ich results from taking an average over a number of rotation lines. This work s been prompted by two considerations ; firstly that the data which can be re- rded upon a spectroscope of normal resolution are averaged in this manner, ondly the desire to avoid working with the spectrum in all its complexity. It ms likely that such methods will be ultimately rejected since it is proving very ficult to take into account simultaneous variations of amount of absorbing material l pressure, although in the important problem of the radiation flux at ground el these difficulties have not yet made themselves felt.

The simplest possible model was worked out by Ladenburg and Reiche (1911), o considered the absorption by a single line of Lorentz shape. This can be taken representative of a band absorption if the line spacing is very much greater than line width. If an average is taken over several lines the apparent absorption at quency ν will be given by

$$A_\nu^* = 1/\delta \int_0^\infty \{1 - \exp\,[-mk\alpha/\pi\{(\nu - \nu_0)^2 + \alpha^2\}]\}d\nu,$$

ich can be integrated in terms of Bessel functions. Although the expression is mplicated it has very simple features. If $km/2\pi\alpha << 1$ then it reduces to

$$A_\nu^* \simeq km/\delta,$$

ich is the linear law of absorption, whilst if $km/2\pi\alpha > 3$

$$A_\nu^* \simeq 2\delta^{-1}(k\alpha m)^{\frac{1}{2}}$$

is is the square-root law of absorption, which many writers claim to have observed th band spectra (*e.g.* Strong 1941 b). The conditions for the observation of the uare-root law are however fairly restrictive. With some rearrangement this ndition may be written

$$A_\nu^* > 8{\cdot}7\alpha/\delta$$

r water vapour at one atmosphere pressure, $\alpha/\delta \simeq 0{\cdot}1$, a comparatively small lue, but even in this case the absorption would have to be 90 per cent. complete the average before the square-root law could be observed, by which time neigh- uring lines would be overlapping, and the theory would not be valid. At lower essures the conditions are more favourable for the observance of the square-root v of absorption, but there is never a large margin before Doppler broadening again kes the theory invalid.

The effect of the mutual interference of lines has been studied by Schnaidt (1939), Elsasser (1942) and Cowling (1950a). Elsasser considered the absorption of an array of precisely similar equally-spaced Lorentz lines, and arrived at the relation

$$A_\nu^* = erf(lm/2)^{\frac{1}{2}}$$

where

$$l = 2\pi k\alpha/\delta^2$$

To obtain this result Elsasser assumed that the centre of each rotation line was fully absorbed. This implies restrictions which are nearly as severe as those which apply to the observance of the square-root law of absorption for a single line. To give a numerical example ; in the case of nitrous oxide at a pressure of one atmosphere, the average absorption must be greater than 80 per cent. before Elsasser's asssumption is valid.

Schnaidt adopted the arbitrary procedure of representing band absorption by the absorption of a single line, cut off at a distance from the line centre equal to half the line spacing. The integration must be performed numerically. There is no good justification for Schnaidt's procedure ; nevertheless Cowling finds that the resulting transmission function is fairly near to the truth for water-vapour absorption.

Cowling has calculated averaged absorptions for water-vapour bands with no simplifying assumptions except that all lines have the same width. He used quantum mechanical information about line positions and relative intensities, but he left the band intensity as a variable ; this is possible since intensity and the amount of absorbing material always occur as a product, and the amount must clearly be kept variable. Although Cowling made calculations in six spectral ranges of the water-vapour rotation band, he concludes that " in atmospheric work complication is avoided, and remarkably little error is involved, if a single absorption curve is used at all wavelengths." Fig. 6 shows Cowling's absorption curve for the water-vapour rotation band compared with those of Elsasser and Schnaidt. The method of plotting makes a shift of the curve along the log (m/m_0) axis equivalent to a change in band intensity, and therefore arbitrary from the point of view of this comparison. Also shown is a very simple empirical function due to Callendar (1941) given by

$$A_\nu^* = m/(m + m_0)$$

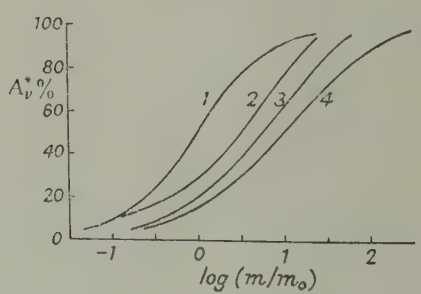

Figure 6. Averaged absorptions, (1) Callendar. (2) Elsasser. (3) Schnaidt. (4) Cowling. (Cowling 1950a).

he conclusion to be drawn is that none of these transmission functions can be ken to represent band absorption in general, although Cowling's presumably gives good representation of water-vapour absorption under conditions of constant essure (one atmosphere in the case of the curve shown in Fig. 5).

6.5. *Pressure effects*

If the absorption follows Elsasser's error-function law or the square-root law, can be seen that the amount of absorbing material and the air pressure (through e line width) are involved as a product. For a gas like carbon dioxide the varia- on of density with height in the atmosphere is practically the same as the variation air pressure with height. For such gases therefore the variations of air pressure will early have as much influence on absorption as the variations of density of the gas, any rate for rays travelling perpendicularly to the earth's surface. If it were true at absorption is only a function of the product pm, then if both $p(z)$ and $\rho(z)$ ry along the path of a ray, it would be possible to work with the product $p(z)\rho(z)$ the only variable. This is the procedure adopted when applying a " pressure rrection " to a radiation chart ; in practice $p(z)$ is raised to some power other an unity on empirical grounds (*cf.* section 3.4.). This treatment is only valid if nes are fully absorbed at their centres, which is a very restrictive condition. It must herefore be recognized that this method of allowing for pressure changes, even if mplicated to the extent suggested by Cowling, has no theoretical backing if absorp- on occurs along a pressure gradient ; it could possibly be as much in error as the ssumption that pressure does not vary with height in the atmosphere.

It is not easy to see how this difficulty can be solved if only averaged data are vailable. A typical averaged quantity which might be measured in the laboratory the integrated absorption of a band

$$A_\tau(p,m) = \int_0^\infty d\nu\{1-\exp[-k_\nu(p)m]\}$$

hile the absorption between a point z_1 and a point z_2 in the atmosphere is given by

$$A_\tau(z_1,z_2) = \int_0^\infty d\nu\{1-\exp[-\int_{z_1}^{z_2}\rho(z)k_\nu(p(z))dz]\}$$

Despite the similarity of these two expressions, there is no known way of going rom one to the other, without the use of simplifying assumptions with unknown epercussions.

The existence of this great stumbling block is the reason that prominence has een given to the fundamentals of band absorption, since if the fine structure of a and is used in all its detail, there is in principle no difficulty in evaluating any adiation problem, although it may well be the task of a calculating machine to do it.

6.6. *Absorption and emission by water vapour*

As has been explained there are great difficulties in the way of using averaged bsorptions unless they have been interpreted in terms of fundamental data ; a iscussion of the majority of laboratory data on atmospheric gases would therefore be

out of place in this review. In the case of water-vapour absorption, much effort has been expended ; recently Cowling (1950a) has examined all the available material for correlation with his theoretical calculations. He uses the experimental data both to demonstrate the accuracy of his absorption function and to find the band intensity, which he leaves as a variable. Unfortunately he has chosen to give his results in the form of the quantity of water vapour which would produce 50 per cent. absorption for each of a large number of wavelength intervals. The actual values of the band intensity are not worked out, so that it is not possible to compare the results with theory, nor is it possible to see whether the results for each range lead to the same value for the band intensity. Cowling's results cover both the 6·3 μ and the rotation bands of water vapour, at room temperature and 220°K, and he also shows how the results will differ at 1, 0·5 and 0·25 atmosphere pressure. This is undoubtedly the best and most comprehensive treatment of water-vapour absorption that has yet been made, and it will probably form the basis of calculations upon the atmosphere for some time to come.

Most radiation charts are based upon the emissivity of water vapour, expressed as a fraction of black-body radiation. The emissivity required is that of an infinite slab rather than that of a column, which corresponds to absorptivity measurements made with parallel radiation. Some writers have suggested that the slab emissivity may be obtained from the corresponding column emissivity with an increase of the amount of water vapour per unit area of surface. The factor by which the water-vapour content has to be increased depends upon the transmission function ; in the case of Elsasser's function it is 1·66, and Robinson (1947) has shown that the experimental information is in favour of this figure. Cowling has calculated emissivities from his absorption data at pressures of 1, 0·5 and 0·25 atmospheres from $m = 10^{-4}$ g cm^{-2} to $m = 10$ g cm^{-2}, and has indicated the effect of temperature upon his figures. He compares his results with the laboratory work upon short paths by Falkenburg (1938) and Elsasser (1942) and with observations made upon the atmosphere by F. A. Brooks (1941) and Robinson (1947) and finds substantial agreement.

7. Radiation and the structure of the lower atmosphere

7.1. *Introduction*

An account of radiation in the earth's atmosphere would be incomplete without some mention of the attempts which have been made to explain the main features of the atmosphere's thermal structure in terms of the emission and absorption of radiation. Such theories have been mainly concerned with the attempt to explain the existence of the tropopause and the lower stratosphere, and they normally do so on the basis that the tropopause is the surface of division between a region where heat transfer is governed by convection (the troposphere) and a region where heat transfer is governed by radiation (the stratosphere). This picture is rather unattractive to the present-day meteorologist, partly because the tropopause has been found to be far more complex than was believed, and partly because the stratosphere has not turned out to be so quiet and stratified as its discoverers supposed. Recent work upon this subject cannot be adequately or relevantly discussed here, but a few main points of interest will be indicated.

Most synoptic meteorologists still tacitly concede that the tropopause represents surface of abrupt physical change. On the other hand it is now quite usual to eak of a multiple tropopause and Normand (1946) when discussing the winter opopause over India, finds the situation so complex that he rejects completely e possibility of any abrupt physical change. The lower stratosphere is certainly ot free from mixing and therefore from convective-heat transfer. Chackett, aneth and Wilson (1950) have measured the ratios of helium, neon, and argon, nitrogen up to 70 km in the atmosphere, and have found no change in relative ncentrations. This indicates effective mixing, although, as Goody (1950a) has inted out, the thermal effects of such mixing may not be large. The neighbourhood the jet stream must, of course, be a region of violent mixing, and various direct easurements have shown that velocity, temperature, and humidity fluctuations e as great in the lower stratosphere as in the upper troposphere.

Clearly the problem of the thermal state of the lower stratosphere will never be xactly solved on the basis of any such simple hypothesis as the existence of radiative quilibrium in this region ; nevertheless many authors have, in the past 40 years, ttempted to treat the problem on this basis. Justification for such attempts is ntained in the obvious statement that if radiative processes, left to themselves, o not demand certain atmospheric motions, then these motions will never be created. should therefore be possible to develop a comprehensive picture of the earth's tmosphere, starting from the case of purely radiative equilibrium and allowing uccessively for each kind of atmospheric motion which the thermal structure will llow and maintain. This is probably an impossible programme, but it is the only nethod of breaking away from the phenomenological approach of section 4, and there s hope that, in some situations, a first approximation to observation may be reached.

Up to the present time the models considered have been symmetrical in hori- ontal planes, taking only vertical convection into account, and neglecting advection. The conclusion reached is always that at sufficient heights, the atmosphere should e in radiative equilibrium, and the point at which this transition takes place is ound to have properties similar to those observed at the tropopause. This may be aken to indicate that, even without advection, a first approximation to observation s reached, but further investigation is desirable, and after the model without advec- ion has been precisely evaluated, the next step will be to consider the various ossible quasi-horizontal motions. At this stage the fundamental causes of the eneral circulation will be under examination, and the investigation will begin to ave common ground with other and perhaps more familiar work.

7.2. *Early work*

In all the early work upon this subject a " grey " absorption spectrum was mployed. This assumption obscures any properties of the atmosphere which may rise from the presence of several absorbing gases, since all absorptions completely verlap each other. It has also been the normal practice to consider water vapour o be the only important absorbing constituent of the atmosphere.

Gold (1909) considered what effect radiation would have upon an atmosphere which was in convective equilibrium throughout. He first calculated the heat balance in a skin layer on the outside of the atmosphere and found that it would

receive more heat by radiation than it would emit ; this layer would therefore heat up and produce a stable temperature discontinuity at its lower boundary. It was also argued that convection would carry heat upwards, and since this layer would already possess an excess of heat, convection would have to cease and radiative equilibrium would be established. In the lower atmosphere on the other hand, there would be a net loss of heat and therefore active convection would take place to supply the deficit from the earth's surface. With the further assumptions that the radiative equilibrium would require an isothermal state with temperature equal to that of the top of the convective layer, Gold calculated the pressure to which the convective layer must extend, in order that radiation balance might be maintained in the radiative layer, to be 0·25 atmosphere.

Emden (1913) and others strongly criticised Gold's theory for many different reasons ; an obvious one being that convection does not necessarily convey heat upwards. Emden calculated the thermal state of the atmosphere under radiative equilibrium and considered its stability ; this is complementary to, but rather more satisfying than Gold's method. A steady mean insolation was assumed, and ground temperature was left as a variable to be determined. Emden found that, under radiative equilibrium, an unstable negative-temperature discontinuity must exist at the earth's surface ; in the atmosphere itself temperature would decrease with height, but would tend to a limiting, non-zero value, which he identified with the stratosphere temperature. Some of the calculated temperatures were :

temperature of the earth's surface	=	309·2°K.
temperature of the lowest layer of the atmosphere	=	289·0°K.
temperature of the highest layer of the atmosphere	=	213·9°K.

Although Emden discussed many aspects of the problem of radiation in the atmosphere in a very able manner, his results suffer from the poor data which he was obliged to employ. One, however, remains of great significance ; he found a lapse rate of temperature at the earth's surface 2·4 times as great as the adiabatic lapse rate, showing that a state of radiative equilibrium would, of its own accord, generate at the earth's surface a convective layer which might reach to a height of several kilometres.

It is not difficult to find fault with Emden's theory. For example it leads to a higher value for the stratosphere temperature over the tropics than over the arctic ; furthermore considerations of water-vapour absorption alone cannot explain an isothermal stratosphere, still less one with positive temperature gradients. Milne (1922), Hergesell (1919), Möller (1941) and others have tried to improve Emden's theory. Linke and Möller (1943) finally come to the conclusion that no approach to radiative equilibrium in the stratosphere is possible without taking account of carbon-dioxide and ozone absorptions. This had already been pointed out by Lindemann (1919), but had been overlooked, presumably because accurate absorption data were not in those days available.

7.3. *The importance of carbon-dioxide and ozone absorptions*

Dobson, Brewer and Cwilong (1946) have given an interesting descriptive account of the way in which the concentrations of water vapour, carbon dioxide

d ozone may influence the thermal state of the lower stratosphere. They first cide whether the various atmospheric absorption bands exert heating or cooling ects upon the lower stratosphere, neglecting back radiation from the stratosphere elf and also direct absorption of solar radiation at these levels. They conclude at the water-vapour and carbon-dioxide bands all try to cool the stratosphere, nile the infra-red ozone bands try to heat it. This difference is caused by the fferences in the equivalent emission temperatures of the troposphere ; the ozone nds are irradiated directly from the earth's surface while for water-vapour and rbon-dioxide bands the source of radiation is in the upper troposphere. If equili- ium is to be preserved, the ozone heating must presumably balance the cooling used by the other gases, and therefore the stratosphere should react to changes ozone concentration in the sense that increase of ozone will lead to higher tem- ratures. Upon this simple argument the authors base their explanation of the incipal thermal features of the stratosphere. Ozone concentrations are greater er the arctic than over the tropics, hence stratosphere temperatures are higher in e arctic ; ozone concentration increases with height, hence the positive temperature adients in the stratosphere ; ozone concentration varies seasonally and, in middle titudes, has a maximum in the spring, which explains the curious change in phase the seasonal variation in temperature which is observed as the tropopause is ossed (and indicates the correct order of magnitude for this variation).

This work presents an interesting and provocative argument, which unfortun- ely suffers because the initial premises are not accurate. For the 15 μ carbon- ioxide band the back radiation from the stratosphere is almost as great as the up- ard flux from the troposphere. Linke and Möller (1943) show that in temperate titudes carbon dioxide will exert a heating effect at the tropopause, and Goody 949) reaches a similar conclusion. In temperate latitudes the main balance is robably between heating by carbon dioxide and cooling by water vapour. levertheless ozone certainly exerts some heating effect, and if solar absorption is ken into account, this heating effect is increased. At greater heights the heating aused by ozone is of course of the greatest importance (Gowan 1947).

7.4. *The thermal equilibrium at the tropopause*

It will be noted that none of the foregoing work has established a criterion for ne limit of the convective layer. Gold stated one which is not acceptable in the ght of modern knowledge, Emden only pointed out that a convective layer will xist, while Dobson, Brewer and Cwilong have only described the direction in hich changes will take place provided that an equilibrium exists. The convective yer must rise at least so far that the lapse rate in the radiative region is less than diabatic and taking this as a boundary condition Goody (1949) showed that an tmosphere in equilibrium with the terrestrial radiation would have an unstable egative-temperature discontinuity just above the convective region. The boundary etween the two regions would therefore rise until the convective layer reached a eight at which the temperature discontinuity at the boundary disappeared. Thus s far as convective disturbances are concerned the absence of a temperature dis- ontinuity between the convective and radiative layers is the condition for equili-

brium between them. There will however be a discontinuous change of temperature gradient at the boundary in the sense that the radiative layer will be the more stable. This equilibrium condition strikingly resembles the description of a normal tropopause, and gives some reason to hope that the identification of the convective layer with the troposphere, and the radiative layer with the stratosphere, may give a first approximation solution to the thermal state of the atmosphere. If the stratosphere is isothermal, this equilibrium condition corresponds to that adopted by Gold for entirely different reasons. To investigate the analogy between this simple model and the earth's atmosphere, Goody made the suggested identification and, using the best existent knowledge about the concentrations of absorbing gases, attempted to calculate how various factors would influence the equilibrium.

Since the troposphere is in convective equilibrium with the earth's surface, increase of surface temperature will lead to an increase of troposphere temperature and will consequently influence the equilibrium position of the tropopause. When this effect is calculated with assumed values for the water-vapour content of the stratosphere, it is found that as ground temperature rises, stratosphere temperature decreases. This is in no way connected with the ozone heating, which is found to be relatively small, but is caused by a complicated interaction of the effects of water vapour and carbon dioxide. This constitutes a possible explanation of the latitude variation of stratosphere temperature, and numerical calculation leads to the correct magnitudes.

The change in stratosphere temperature which would accompany a seasonal change in ozone concentration may be calculated, and the result agrees in magnitude with the value assumed by Dobson, Brewer and Cwilong. Nevertheless Goody (1950b) finds a serious discrepancy between the predicted and observed phase relations. If, however, direct heating by solar radiation near the tropopause is taken into account, the observed magnitudes and phases of the seasonal variations of stratosphere temperature in the tropics, arctic, and temperate latitudes (Möller 1938), can be qualitatively understood.

When the temperature gradients of the lower stratosphere are examined, it appears that the argument used by Dobson, Brewer, and Cwilong to account for the nearly isothermal structure is capable of generalisation. Provided that more than one absorbing gas takes part in the thermal equilibrium an effect such as that described by these authors probably exists. In temperate latitudes the participating gases are probably carbon dioxide and water vapour, and order-of-magnitude calculations show that the stratosphere could be approximately isothermal if it were in radiative equilibrium.

7.5. *Conclusion*

The work described in this section is based upon the use of simplified models of the atmosphere. While the usefulness of such research is now probably exhausted some simple ideas have been established which will be helpful to future workers. The line of attack most likely to improve our knowledge of the thermal state of the atmosphere is the development of numerical or graphical methods similar to, but more precise than, the radiation charts which are suitable for low-level investigation.

8. Scattering

8.1. *Introduction*

The important influence of scattering upon the state of the atmosphere is that it partially degrades the solar beam into a diffuse flux of energy, and returns a considerable proportion unused into space. Scattering centres may be classified into :

(a) Molecules or particles much smaller than the wavelength of light ; their scattering is often referred to as Rayleigh scattering.

(b) Dielectric spheres ; this is a model used to treat the scattering from liquid drops.

(c) Solid particles of arbitrary shape and size ; the problem of scattering from dust particles or of diffuse reflection from solid surfaces.

Van de Hulst (1949) has made a valuable contribution to meteorological literature with his review article on " Scattering in the atmospheres of the earth and planets," to which the reader is referred for more extensive information.

8.2. *Measurements of scattering*

It is possible to measure either the depletion of the incident beam, or the intensity and direction of the scattered radiation. Provided that the radiation can be considered monochromatic, the depletion is governed by Beer's Law. The extinction coefficient, unlike gaseous absorption coefficients, is a slowly-varying function of frequency, and therefore the conditions which must be satisfied before the beam may be considered to be monochromatic are not restrictive. The extinction coefficient can be measured by simple observations upon depletion (Sinclair 1947), provided that it is fairly large. If the extinction coefficient is small, path lengths may be necessary which can only be obtained outside the laboratory. Measurements taken in the open during the day are complicated by the sunlight which may be scattered into the beam, giving the air an inherent brightness per unit length. The apparent brightness of a black object at distance z is then

$$b_\nu(z) = s_\nu \beta_\nu^{-1}[1 - \exp(-\beta_\nu z)]$$

and the contrast between the black object and the horizon is given by,

$$b_\nu(z)/b_\nu(\infty) = 1 - \exp(-\beta_\nu z)$$

This relation, usually called after Koschmieder, can be used to measure the extinction coefficient. Difficulties are many. The test object has to be large enough to avoid troublesome edge effects (Fry, Bridgman and Ellerbrock 1947), allowance must be made for the intrinsic brightness of the black object (Dessens and Kastler 1944) ; and there is always doubt about the homogeneity of the atmosphere.

At a sufficient distance the brightness contrast between the horizon and the test object will become so small that the object will be invisible. It seems to be generally accepted that a contrast of 2 per cent. is the limit that the eye can detect, and the distance at which this contrast is observed will be related to the visual extinction coefficient by the equation

$$\beta v \simeq 4{\cdot}0$$

The distance v defined by this equation is the visual range, and it is therefore possible to give visibility in terms of the extinction coefficient, and *vice versa*. The validity of this relation has been tested by many workers (*e.g.* Hulburt 1941), and it is found to be obeyed extremely well. The extinction coefficient for pure air can be calculated and is found to be 0·014 km^{-1}, which corresponds to a visual range of 280 km. Such visibilities never occur at ground level, from which it may be concluded that scattering by dust and drops is always of importance. Even on a very clear day (International Code Number 8) the extinction coefficient is between 5 and 15 times its clear-air value.

If the scattered light, rather than the depletion of the beam, is to be measured, complicated angular and polarisation relationships come into consideration. Angular distributions have been measured, using artificial beams (searchlights) and the sun as sources, by Hulburt (1941), Reeger and Siedentopf (1946), Bricard (1940) and Bullrich (1947). Observations upon the sun take the form of a measurement of sky brightness. Over most of the sky scattering is a very complicated process, although two fairly simple cases exist. Scattering by particles large compared with the wavelength of the light is largely in a forward direction, while molecular scattering is more diffuse. Sky brightness near to the sun therefore is mainly caused by non-molecular scattering which gives rise to the bright ring or aureole around it. If the aureole is not too bright it is not difficult to derive the angular scattering coefficient from measurements of its angular intensity (Van de Hulst 1949). Another simple case is the horizon brightness, since it is reasonable to assume that the path is the same in whichever direction the horizon is viewed, and the only variable will therefore be the angle of incidence of the solar beam. Hulburt (1941) has obtained scattering diagrams from measurements of horizon brightness, although the interpretation is somewhat complicated by the double scattering which takes place.

Direct observation of the sun through mist or cloud is complicated to such an extent by double scattering that it is virtually impossible to interpret the results precisely. Neiburger (1949) has however shown from measurements of the scattering of sunlight by cirrus cloud that the scattering diagram must have a strong forward peak, as is expected from theoretical considerations.

8.3. *Molecular scattering*

Although molecular scattering is swamped by particle scattering in the lowest 2 or 3 km of the atmosphere, it is nevertheless responsible for a large proportion of the observed sky light. Any volume element in the sky receives one fifth of its solar radiation after single scattering by molecules, and the process is therefore of energetic importance.

Rayleigh's theory applies to spherical scattering centres which are much smaller than the wavelength of the light used, and it leads to the following relation for the extinction coefficient

$$\beta_\lambda = 32\pi^3(n-1)^2/3N\lambda^4$$

Features of Rayleigh's law which call for confirmation are the dependence of extinction on the fourth power of wavelength and the correctness of the derived value

of Avogadro's number. The former requires spectroscopic measurements of skylight in spectral regions which are free from absorption bands and under conditions where dust and water droplets are not present. Numerous measurements of this kind have been made, and Dufay (1940) and Götz and Schönmann (1948) conclude that there is excellent agreement with Rayleigh's law. This conclusion is not sacrosanct however, since Tousey and Hulburt (1947) and Duclaux (1946) both report contradictory evidence.

8.4. *Liquid-drop scattering*

The electromagnetic theory of Mie is the basis of all recent work on the scattering of light by dielectric spheres. The extinction coefficient is usually given in the following form

$$\beta_\lambda = \pi r^2 N\, E(2\pi r/\lambda, n),$$

$\pi r^2 N$ being the geometric cross section of the drops in a layer 1 cm thick, and the complicated function $E(2\pi r/\lambda, n)$ the ratio of the scattering and geometric cross sections. This function has been computed for non-absorbing spheres with $n = 1{\cdot}33$ (appropriate to pure water) by Stratton and Houghton (1931) and by Houghton and Chalker (1949). Other workers (*e.g.* Sinclair 1947) have made computations for values of n up to 2·0 and values of $2\pi r/\lambda$ up to 12.

Fig. 7 shows the results of a number of calculations which have been made. The curves have a fine structure which is shown in the case $n = 1{\cdot}50$, but this is not an important feature. For small values of $2\pi r(n-1)/\lambda$ Rayleigh's law is obtained, as must be expected. There is a maximum of extinction for $2\pi r(n-1)/\lambda = 2$, and there are subsidiary maxima thereafter. For values of $2\pi r/\lambda$ greater than 150, E reaches a limiting value of 2. From geometric optics this limiting value should be unity; however according to Sinclair (1947) the value

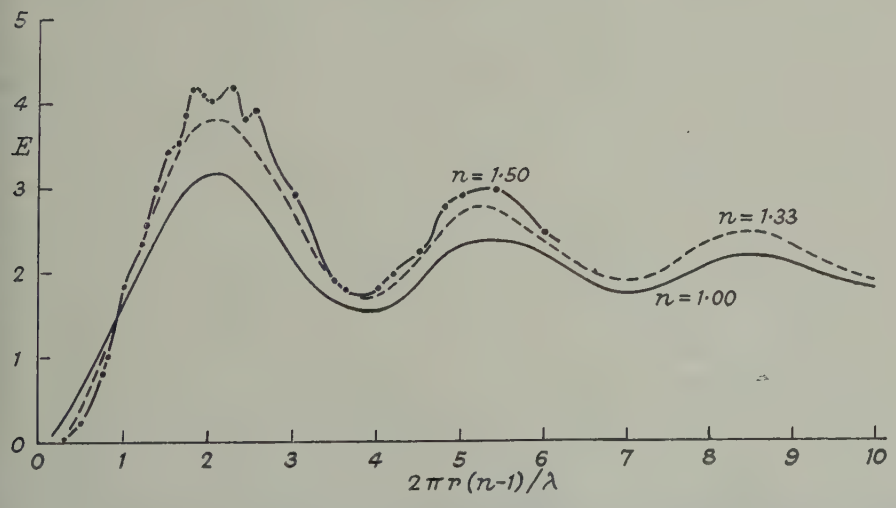

Figure 7. The ratio of optical to geometric scattering cross section for dielectric spheres, as a function of size, wavelength, and refractive index. The curve marked $n = 1{\cdot}00$, for spheres of refractive index very close to 1, is computed from an exact formula given by Mie (van de Hulst 1949).

2 is undoubtedly correct, and may be verified experimentally. For very large spheres, E may be divided into two terms ; one corresponds to the case of geometric optics, the second is a diffraction effect which scatters light into a small solid angle almost along the incident beam. These two terms make equal contributions to the scattering cross section.

The curves of Fig. 7 have been verified experimentally to some extent. Sinclair (1947) has investigated the case of $n = 1{\cdot}44$ by using an artificial fog of stearic-acid droplets. The existence of scattering maxima explains some curious observations which have been made of the wavelength dependence of atmospheric extinction coefficients. Typical of such measurements are those of Coleman and Rosenberger (1950) who found that in the visible spectrum, extinction varied inversely with wavelength. Vassy and Vassy (1939) find extinction which is proportional to wavelength and even a maximum of extinction at one particular wavelength. Van de Hulst (1949) finds it possible to explain all these measurements by assuming various reasonable drop sizes for the scattering particles.

The size of a hygroscopic droplet will vary with humidity, and there should therefore be a correlation between visibility and humidity. Wright (1939) has constructed a theory based upon the assumption that the hygroscopic nuclei are particles of sodium chloride, a suggestion directly confirmed by Dessens (1949), and Hilsum (1950) adopts Wright's suggestion when interpreting measurements of the percentage transmission at $3{\cdot}61\ \mu$ and $2{\cdot}18\ \mu$ as a function of the transmission at $0{\cdot}61\ \mu$. Making assumptions about the drop-size distribution and the variation of refractive index as the drop size varies, he shows that Mie's theory gives a good account of the observations.

8.5. *Information obtainable from scattering experiments*

Knowledge of scattering processes can be made to yield information of interest to other branches of meteorology. Thus information upon drop sizes and the number of particles per unit volume can be deduced if a sufficiently good experiment can be planned. An example is given by aufm Kampe's attempt (1950) to measure the liquid-water content of clouds by means of a scattering experiment. Neuberger (1950) makes a strong plea for careful observations of the position of Arago's neutral point in order to give information upon scattering centres in the atmosphere. Arago's point is one of three in the sun's meridian where there is no polarisation, and it owes its existence to a mixture of scattered sunlight and doubly scattered light from the earth and atmosphere. The position of the point is therefore sensitive to atmospheric scattering. Neuberger used measurements of the Arago point to show that dust from the Katmai volcano lingered in the air for several years, and that the larger particles fell to the ground first, as might be expected. He also claims that there is a shift in the Arago point an hour or two before rain falls.

Measurements of molecular scattering can lead to estimates of the air density in the upper atmosphere. Such measurements are best made at twilight, when the solar radiation does not pass through the lower atmosphere and the upper layers are illuminated with grazing incidence. Grandmontagne (1941) has made photometric observations on the twilight sky, and Gauzit and Grandmontagne (1941)

have interpreted these measurements in terms of the change in density of the atmosphere with height. It was found that at 57 $\pm$ 7 km a change in temperature gradient of 16·5°C km^{-1} occurred, which agrees well with information gained from other sources. Van de Hulst believes that this kind of experiment could be made fairly precise if an organised effort were made with many observers, and that it represents one of the best potential methods of measuring temperature in the high atmosphere.

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The meridional eddy flux of energy

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SUMMARY

An investigation of a large number of upper-air soundings over regions of North America shows that the eddy transport of sensible heat is poleward in the mid-latitude troposphere, reversing and becoming equatorward in the stratosphere. The eddy transport of sensible heat reaches its maximum in middle latitudes, decreasing both poleward and equatorward. It is shown wherein these results differ from those recently presented by Priestley. The transport of sensible heat has a high seasonal variation, being much stronger in winter than in summer. There are indications that at very low latitudes the eddy sensible-heat transport is directed equatorward.

The eddy transport of latent energy is described by means of the same soundings and is found to be poleward throughout middle and high latitudes with a maximum at the ground and decreasing with altitude. The principal seasonal variation in the transport of latent energy appears to be the northward shift of the mid-latitude maximum from winter to summer. This maximum is not very pronounced, the transport in both seasons being relatively constant through middle latitudes.

1. INTRODUCTION

A description of the character of the meridional flux of energy in the atmosphere is recognized to be of fundamental importance for a better understanding of the mode of operation of the general circulation. That a description of so basic a process in the atmosphere has not been clearly established can be attributed directly to the lack of observational material on a hemispherical basis, and to the tremendous task of reducing whatever data are available to comprehensible form. A method for the evaluation of certain components of the meridional energy flux has recently been reported by Priestley (1949). A study of the meridional flux of energy using a method similar to Priestley's but considerably more extensive in space, is described below.

2. SCOPE

To place the scope of this investigation in perspective it is necessary to consider the general continuity requirements of the global energy balance. Because there exists a difference between the net heating of the atmosphere in equatorial and polar regions, there must exist in the atmosphere a transport of energy directed meridionally from low to high latitudes. Starr (1949) has shown and indeed it may be inferred from general considerations that the total poleward energy flux may be closely approximated by an expression of the form

$$T^* \simeq \int_{z=0}^{z=\infty} \oint (c_p T + ql + \Phi)\rho v \, dx \, dz \quad . \quad . \quad . \quad (1)$$

* This investigation was made possible through support extended by the Geophysical Research Directorate of the U.S.A.F. Cambridge Research Laboratories.

where c_p is the specific heat of air at constant pressure, T is the temperature, q is the specific humidity, l is the latent heat of water vapour, Φ is the potential energy per unit mass, ρ is the density, v is the meridional component of velocity considered positive when the wind is from the south, dx is an element of longitudinal distance and dz an element of vertical distance. The integration is to be performed over the entire vertical extent of the atmosphere and completely around the latitude circle. Eq. (1) has the simple interpretation that the poleward flux of energy is accomplished mainly as a flux of (a) sensible heat, (b) latent energy of water vapour, and (c) potential energy.

One way in which the terms in Eq. (1) may be further examined is as follows. We may write that

$$\rho v = \left[\rho v\right] + (\rho v)'$$

where $\left[\rho v\right]$ is the average of ρv along the entire length of a closed latitude circle and $(\rho v)'$ is the deviation from this average. It is clear that the identical vanishing of $\left[\rho v\right]$ implies the absence of meridional circulations, while its failure to vanish implies the existence of such circulations. Here we neglect all topographic inequalities of the earth's surface. Since Φ is constant along a latitude circle at any given elevation and $\left[(\rho v)'\right]$ is zero, Eq. (1) may be rewritten as

$$T^* \simeq \int_{z=0}^{z=\infty} \oint (c_p T + ql)(\rho v)' dx dz + \int_{z=0}^{z=\infty} \oint (c_p T + ql + \Phi)\left[\rho v\right] dx dz \quad . \quad . \quad (2)$$

where the last integral now represents the contribution of mean meridional circulations of the Hadley type to the poleward energy-flux. The first integral in the right hand member of Eq. (2) represents the contribution of horizontal eddy-processes to the poleward energy-flux. It is the magnitude of these processes which can be estimated with available radio observations of wind, temperature and moisture content.

3. Procedure

Following Priestley we may write that the total meridional flux of any quantity across a complete circle of latitude is given very closely by,

$$\oint dx \int_{z=0}^{z=\infty} \rho F v dz \quad . \quad . \quad . \quad . \quad (3)$$

where F is the amount of the quantity per unit mass of the atmosphere. The integration is to be performed through the entire vertical extent of the atmosphere, and around a complete latitude circle. For practical purposes, it is more convenient to write this integral as

$$\frac{1}{g} \oint dx \int_0^{p_0} F v dp \quad . \quad . \quad . \quad . \quad (4)$$

where use has been made of the hydrostatic equation, and where p_0 is the sea level

pressure. Because observations around a complete circle of latitude are not available it is impossible to compute this transport observationally. However, we may obtain highly useful information by considering the flux over a limited portion of the circle which is given for a unit distance along a latitude circle by

$$\frac{1}{g}\int_0^{p_0} Fv\,dp \qquad . \quad . \quad . \quad . \quad (5)$$

In expression (5) the sea level pressure p_0 is not constant. However, p_0 varies by only a small fraction of its absolute magnitude. In addition, in the computation of such a term over a large sample, p_0 may be considered as a constant if there is little correlation between Fv and p_0. We shall assume that the quantities used in the following analysis, the temperature and the mixing ratio multiplied by the meridional component of the velocity, are uncorrelated with the sea level pressure. Therefore, we shall treat p_0 as a constant in what follows.

In the following analysis it will be necessary to make use of both space and time means and deviations from these means. For convenience in notation, a bar is used to denote a time mean, a bracket a space mean. Primes denote deviations from the means and the subscripts x and t are used to denote the nature of the deviation. Thus we may write $F = \bar{F} + F_t'$ or $F = \left[F\right] + F_x'$ where F is the actual value of the quantity, F_t' is the deviation from the time mean and F_x' is the deviation from the space mean. In a similar fashion we may define a mean of F in both space and time as $\left[\bar{F}\right]$.

The mean meridional flux of F over both space and time, $\left[\overline{vF}\right]$ is given by

$$\left[\overline{vF}\right] = \left[\bar{v}\right]\left[\bar{F}\right] + \left[\overline{v'_{tx}F'_{tx}}\right] \qquad . \quad . \quad . \quad (6)$$

which is derived by taking the mean of the product $v = \left[\bar{v}\right] + v'_{tx}$ and $F = \left[\bar{F}\right] + F'_{tx}$. The mean flux of F is then composed of two terms of distinct physical significance. Following Priestley's terminology, let us call $\left[\bar{v}\right]\left[\bar{F}\right]$ the advective flux. This is the flux of the quantity F at any given level in the atmosphere which is associated with a non-zero mean rate of flow $\left[\bar{v}\right]$ at the level in question. The second term $\left[\overline{v'_{tx}F'_{tx}}\right]$ is the mean eddy flux and depends upon the existence of a correlation between v and F at any given level.

It can be shown that the mean eddy flux may be further analysed into two terms of different physical significance. This is easily done by adding to and subtracting from the left hand side of the Eq. (6) the term $\left[\bar{v}\,\bar{F}\right]$. After slight rearrangement it can be seen that

$$\left[v'_{tx}F'_{tx}\right] = \left[\overline{v'_tF'_t}\right] + \left[\bar{v}'_x\bar{F}'_x\right] \qquad . \quad . \quad . \quad (7)$$

in which the terms on the right hand side have the following meanings :

(a) $\left[\overline{v'_tF'_t}\right]$ represents the space mean of the local eddy flux. This is the average over space of the eddy flux at individual stations due to the correlation in time of v and F.

(b) $\left[\bar{v}'_x \bar{F}'_x\right]$ represents the standing eddy flux. This is the flux due to the correlation in space between the time means of F and v over a given period of time. In other words, this is the portion of the flux due to the semi-permanent features of the atmosphere.

Ideally, if we wished to compute the total northward eddy flux for a given period from actual wind-data, we should desire stations located around a complete circle of latitude. The more closely we could approximate a continuous distribution of stations, the more accurately we could obtain the eddy transport across that latitude circle. We could then compute the total eddy flux once or twice daily, and obtain the mean flux for any given period. In the absence of such a distribution of observing stations the existing correlation between v and F may be approximated by the use of a large number of stations distributed over a limited portion of the latitude circle. Because of the nature of the available data, all of the computations which follow have been made with reference to constant pressure surfaces.

In the case of the transport of sensible heat, $F = c_p T$. The mean eddy flux of sensible heat is then given by

$$\frac{c_p}{g}\int_0^{p_0}\left(\left[\overline{vT}\right]-\left[\bar{v}\right]\left[\bar{T}\right]\right)dp \quad . \quad . \quad . \quad . \quad (8)$$

The mean eddy flux of latent heat may be written similarly as

$$\frac{l}{g}\int_0^{p_0}\left(\left[\overline{vq}\right]-\left[\bar{v}\right]\left[\bar{q}\right]\right)dp \quad . \quad . \quad . \quad . \quad (9)$$

where l is the latent heat of water vapour per unit mass taken as 600 cal gm^{-1}, and q is the specific humidity.

In his interesting paper, "Heat transport and zonal stress between latitudes," Priestley (1949) selected a single station, Larkhill, England, and performed such computations over a period of two years. However, since he used only a single station, he was able to evaluate only the local eddy flux term.

He found that "the eddy flow of sensible heat at some levels is from regions of low towards regions of higher mean temperature and that the eddy fluxes of heat in sensible and latent forms are approximately equal." These conclusions must be modified however when the standing eddy flux of these quantities is also considered.

In order to include a measure of the standing eddy flux it is necessary to take into account some of the semi-permanent features of the general circulation. The investigation was extended to include subtropical and polar regions so as to be able to determine the variation of such eddy transports with latitude. The region of North America and the adjacent regions of the Atlantic Ocean were chosen for study. In order to eliminate any bias toward cloudy or clear weather only stations which take radiosonde and radio wind observations were selected*.

The months of February and August 1949 were selected for study. The upper-air data were taken from the *Daily upper air bulletin* published by the U.S. Weather

* The observing stations were distributed as follows : South of 30°N. — 6 ; 30° to 40°N. — 16 ; 40° to 50°N. — 14 ; 50° to 60°N. — 6 ; 60° to 70°N. — 6 ; 70° to 80°N. — 4.

Bureau. The data were broken down into three groups : (a) Feb. 0300 GMT, 1949, (b) Feb. 1500 GMT, 1949, (c) Aug. 1500 GMT, 1949. Only those soundings which reached a height of 300 mb were used in the computations. It was felt that such a restriction would enable more accurate comparison of low-level and high-level transports*.

The data were recorded at each station at each of the standard pressure levels : 1000, 850, 700, 500, 300, 200, and 100 mb. The products vT and vq were formed for each level and each time category for each latitude belt. Then $[\overline{vT}]$, $[\overline{vq}]$, $[\bar{v}]$, $[\overline{T}]$, $[\bar{q}]$ were computed by a simple averaging process†. The subtractions $[\overline{vT}] - [\bar{v}][\overline{T}]$ and $[\overline{vq}] - [\bar{v}][\bar{q}]$ were performed for each level, time category, and latitude belt. These were then converted into the mean flux in cal sec^{-1} per mb of pressure difference in the vertical per cm of longitude. Because of the unrepresentativeness of observations taken near the surface (due to radiation and local topographic effects) the values of the transports of energy obtained for the 1000 mb surface were not used. Instead, in the integrations through height which were made in order to obtain the mean total eddy transport throughout the atmosphere, it was assumed that the transports at 850, 700, 500, 300, 200, and 100 mb were representative of the transports between the surface and 800 mb, 800 and 600 mb, 600 and 400 mb, 400 and 250 mb, 250 and 150 mb and 150 and 50 mb, respectively. It was assumed that the transports above 50 mb could be neglected.

The results of this investigation are presented in the form of a series of diagrams and tables. A short discussion of the nature of these data is necessary for proper interpretation. The data may be considered to be of two types, (a) the eddy transports of either sensible heat or latent energy or a combination of both at given levels and latitudes, and (b) the total eddy transports of sensible heat or latent energy or a combination of both, by which is meant the value of the transport after integration through height. It must be remembered that the results presented are valid only for regions of North America and only for the specific times for which they were computed.

Tables I and II present the total eddy transports of the two energy forms, by which is meant the values of the eddy transports after integration through height, across various latitude circles. This was done under the assumption that these transports are representative of the transports around a complete circle of latitude. The values of these total transports are given in units of 10^{11} cal sec^{-1} across a complete circle of latitude.

* Such a restriction undoubtedly has some effect upon the magnitudes of the energy transports because of the selectivity of radio wind observations. Dr. E. N. Lorenz (1950) has found in an investigation into this selectivity, a pronounced tendency for radio wind observations to be biased according to the wind speed. In periods of strong winds there is a decided tendency for balloons to be blown out of range or below the angle specified in observing manuals below which observations are considered unreliable and hence not reported. The effect of such a bias is to minimize the magnitude of the energy transports. Hence the magnitudes of the energy transports reported herein may lean in the direction of under-estimation rather than over-estimation.

† Such a simple averaging process would be entirely legitimate were the selected stations evenly spaced along latitude circles. However, the stations are irregularly spaced, and hence such an averaging process introduced a bias because each station is weighted equally. The results therefore may be biased in favour of the character of the eddy transport in those regions where the stations are more closely spaced. In order to correct this bias, proper weighting factors would have to be introduced. However, since the stations are irregularly spaced in both latitude and longitude, it is not clear how a proper weighting procedure could be devised.

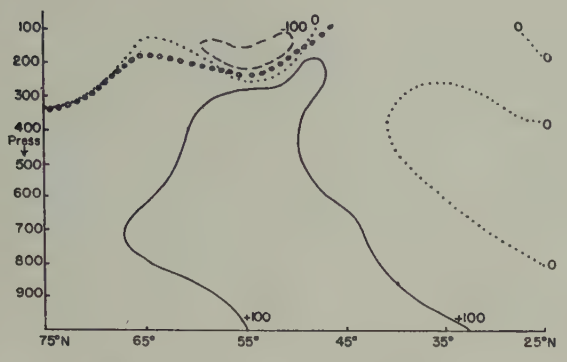

Figure 1. Transport of sensible heat computed from soundings for August 1949, 1500 GMT.

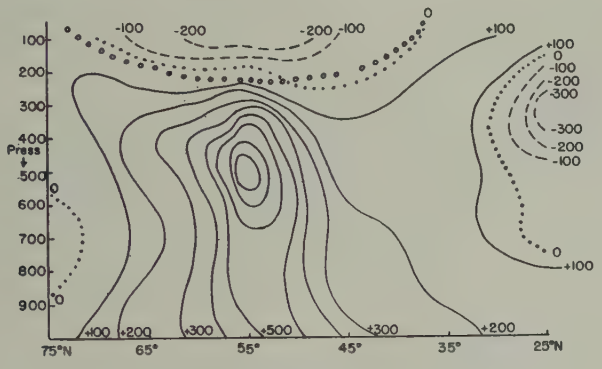

Figure 2. Transport of sensible heat computed from soundings for February 1949, 0300 GMT + 1500 GMT.

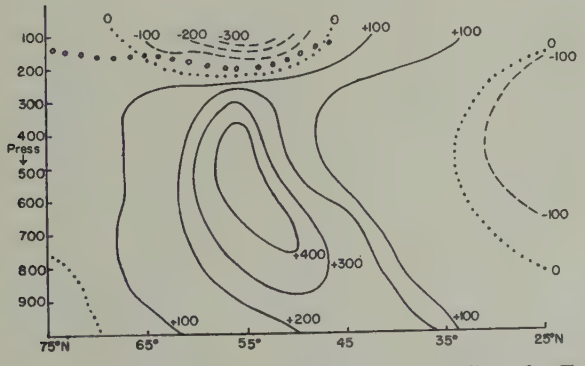

Figure 3. Transport of sensible heat computed from soundings for February 1949, 1500 GMT + August 1949, 1500 GMT.

← Latitude

Figs. 1-3. The mean eddy transport of sensible heat in the atmosphere over regions of North America. Isolines represent the eddy transport in units of cal/sec x 10^7 per degree of longitude per mb pressure difference in the vertical. Solid isolines indicate a northward transport ; dashed isolines a southward transport. The isoline of zero transport is represented by dots. The height of the mean tropopause for the cases examined is indicated by open dots.

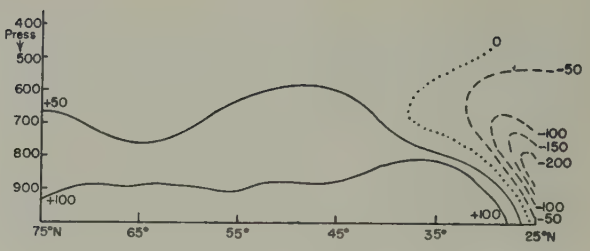

Figure 4. Latent-energy transport computed up to 500 mb from soundings for August 1949, 1500 GMT.

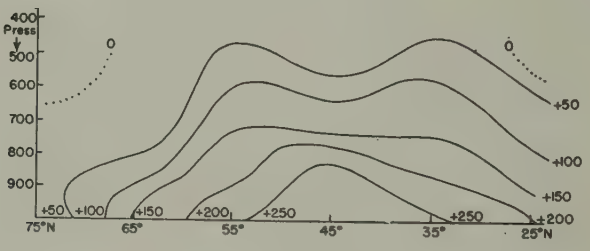

Figure 5. Latent-energy transport computed up to 500 mb from soundings for February 1949, 0300 GMT + 1500 GMT.

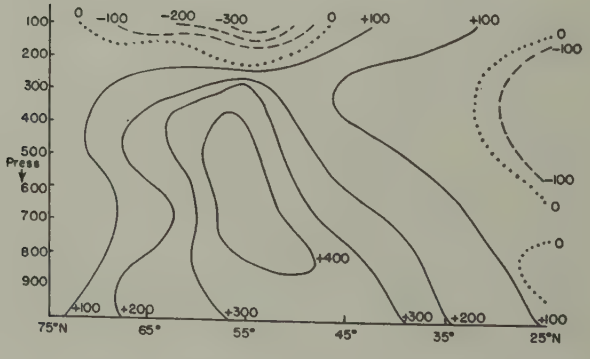

Figure 6. Transport of energy as the sum of the latent energy and sensible heat computed from soundings for February 1949, 1500 GMT + August 1949, 1500 GMT.

← Latitude

Figs. 4-6. The mean eddy latent-energy transport and the mean eddy transport of the sum of the latent energy and sensible heat in the atmosphere over regions of North America. Isolines represent eddy energy transport in units of cal/sec x 10^7 per degree of longitude per mb pressure difference in the vertical. Solid isolines indicate a northward transport ; dashed lines a southward transport. The isoline of zero transport is represented by dots.

The eddy energy transports are presented in the form of diagrams of isolines f transport in Figs. 1-6. These diagrams, drawn in a coordinate system in which he vertical coordinate is the pressure on a linear scale, and in which the isolines of ransport are in cal sec^{-1} *per mb pressure difference in the vertical* per degree f longitude at the given pressure levels. These diagrams have the advantage that he eddy energy transports are presented as a function of the atmospheric mass. To prevent erroneous interpretations of such diagrams, a single sample diagram ig. 7, is presented in which the vertical coordinate is geometrical height on a linear cale, and the isolines of transport are in cal sec^{-1} *per cm of height* per degree of ongitude at specific geometric heights.

4. Interpretation of results

The pictures of the eddy energy transports computed from actual wind data ver regions of North America, as outlined above, reveal many interesting features see Figs. 1, 2, and 3).

The sensible-heat transport during February 1949, which has been taken as ypical of a winter season, is northward within the troposphere everywhere north f 25°N., and southward in the stratosphere in mid-latitudes. This is well brought ut in Fig. 2 where the height of the mean tropopause for February 1949 based upon emperature observations has been sketched in. It can be seen that there is remarkble agreement between the tropopause position and the level of reversal of the irection of the eddy transport of sensible heat.

A noticeable feature of the eddy transport of sensible heat during this month s the location of the latitude of maximum transport at 55°N. One may not draw he conclusion that this represents the winter mean latitude of maximum transport ecause of the limited amount of data treated.

At the latitude of maximum intensity, the transport per mb of pressure difference eaches its maximum at some level above the ground, in this case at 500 mb. In the egions south of the latitude of maximum transport, the eddy transport per mb of ressure difference is at a maximum in the lowest layers of the atmosphere.

Some mention should be made about the indications of a southward eddy ransport of sensible heat in the middle troposphere near 25°N. However, it should e remembered that at such low latitudes only a small section of the latitude circle as been sampled and the standing eddy-flux term may not be adequately represented n the data from a limited region south of the United States.

During August 1949 (see Fig. 1) the same characteristics evident during the winter season are again present. The transports are northward but much weaker hrough most of the mid-latitude troposphere, reversing in the stratosphere. Again he coincidence of the tropopause with the level of the reversal of the transport irection may be noted. The latitude of maximum transport is at 55°N. with the evel of maximum intensity again located at the 500 mb level. In lower middleatitudes the transport is greatest near the ground. At low latitudes there is marked difference from winter to summer, in that the region of southward transport of ensible heat extends farther to the north.

With the present data the closest approximation to a picture of a typical yearly mean may be computed by combining the eddy transport for August, 1500 GMT

and February, 1500 GMT, Fig. 3. The characteristics of such a mean picture are again :

(a) Northward eddy flux of sensible heat in the troposphere.
(b) Latitude of maximum transport at 55°N.
(c) Level of maximum transport at 500 mb.
(d) Southward transport in the stratosphere.
(e) Southward transport at lowest latitudes.

In our discussion to this point, the eddy transports of sensible heat have been referred to constant pressure levels and discussed in terms of an eddy transport per mb of pressure difference. Though this method of description has many advantages in that it permits easy association of such transports within given masses of the atmosphere, it is of interest to relate such transports to a more readily pictured geometrical coordinate system with the transports given per unit height. It is, of course, obvious that such a conversion to a geometrical coordinate system will greatly emphasize the transports at the lower geometrical levels at the expense of the transports at the higher geometrical levels.

To accomplish this transformation, the eddy transport of sensible heat at each level was assigned to the geometric height above sea level as given by the NACA standard atmosphere. Although it is realized that the elevation of an isobaric surface can in no way be considered constant, the transformation is of sufficient accuracy to depict the desired features. In converting the eddy transports of sensible heat per mb of pressure difference in the vertical into the eddy transport per unit height, the mean temperature at each pressure level was used.

The diagram of the eddy transport of sensible heat in the new coordinate system for August 1949, 1500 GMT plus February 1949, 1500 GMT is shown in Fig. 7. This may be compared with Fig. 3 in which the same data are presented in the former coordinate system. The major difference between the two figures is the

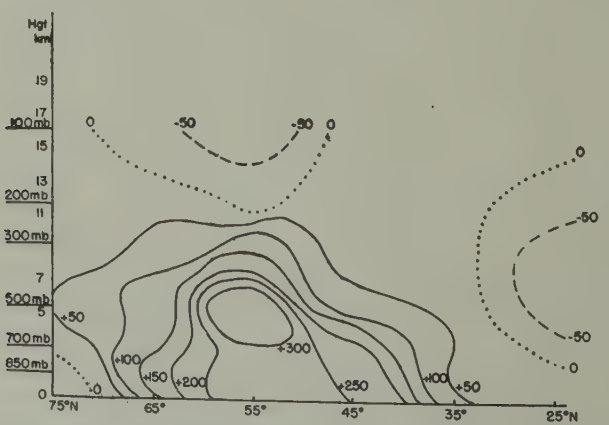

Figure 7. The mean eddy transport of sensible heat in the atmosphere over regions of North America for August 1949, 1500 GMT and February 1949, 1500 GMT described by means of isolines of transport in a geometrical coordinate system with latitude and elevation as the coordinates. The eddy transport is in units of 10^4 cal/sec per degree of longitude per unit height. Solid isolines indicate a northward transport, dashed isolines a southward transport. The isoline of zero transport is indicated by dots. The preponderance of the eddy transport in the lowest layers of the troposhere is emphasized.

concentration of the eddy transport of sensible heat in the lowest geometrical levels of the atmosphere in Fig. 7. The strong maximum of transport per mb of pressure difference at the 500 mb surface in Fig. 3 is considerably reduced in magnitude relative to the transports at lower layers in Fig. 7.

The eddy transport of latent-heat has a maximum in the lowest layers and decreases with height at all latitudes. During the winter season, as typified by the February data (see Fig. 5), the latent-heat transport has a slight double maximum at 35° and 55°N. and in the lowest layers. The reality of this double maximum as a mean hemispherical condition is questioned and is probably attributable to the location of the selected stations. During the summer, as typified by the August data (see Fig. 4), the maximum is to the north and in the lowest layers. An interesting feature is the southward flow in the very low latitudes.

In the final analysis, the sum of the transports of latent energy and sensible heat must be of fundamental significance for the global energy balance. The typical yearly-mean picture of the transport of the sum of these energy quantities, as computed from the 1500 GMT data for February and August 1949, appears in Fig. 6. The major characteristics may be stated as follows :

(a) Latitude of maximum eddy flux — 55°N.
(b) Level of maximum eddy flux at latitude of maximum eddy flux — 500 mb.
(c) Northward transport in mid-latitude troposphere.
(d) Southward transport in mid-latitude stratosphere.
(e) Southward transport in low-latitude troposphere.
(f) Decrease of transport with height in lower mid-latitudes.

The eddy transports of the various forms of energy may be integrated through height, under the assumption that such transports computed for regions of North America are representative of the transport across a complete circle of latitude. The latitude of maximum eddy transport of sensible heat during both winter and summer, as typified by the North American observed wind data, Table I, occurs at 55°N.

TABLE I. The total transport of (a) sensible heat and (b) latent heat, integrated vertically to the top of the atmosphere, transformed into the total transport across a latitude circle under the assumption that the data for North America are representative for the entire hemisphere. Values given for various latitudes and times of year. Units in cal/sec $\times 10^{11}$. F = Feb., A = Aug.

Lat. °N.	F0300	F1500	A1500	F0300 + F1500	F1500 + A1500
			(a) Sensible heat		
25	+ 399·3	− 3305·1	− 47·2	− 1452·9	− 677·1
35	+ 4875·7	+ 4303·4	+ 134·5	+ 4578·9	+ 2230·4
45	+ 3885·6	+ 5590·6	+ 3130·0	+ 4738·8	+ 4704·9
55	+ 21756·1	+ 10452·7	+ 4051·0	+ 16192·6	+ 7352·0
65	+ 3300·6	+ 5095·4	+ 1967·2	+ 5191·7	+ 3841·4
75	+ 743·6	+ 1048·3	+ 1033·2	+ 667·2	+ 1039·5
			(b) Latent heat		
25	+ 1074·5	+ 1764·2	− 2337·7	+ 1401·2	− 290·4
35	+ 3332·5	+ 2381·3	+ 1095·5	+ 2853·6	+ 1745·0
45	+ 1450·0	+ 1318·8	+ 1630·1	+ 1384·4	+ 1477·3
55	+ 2825·9	+ 2097·6	+ 1534·3	+ 2464·0	+ 1818·3
65	+ 588·1	+ 919·4	+ 1024·1	+ 757·1	+ 868·7
75	+ 145.6	+ 16·6	+ 1354·1	+ 108·2	+ 634·4

The average of the data for February 1949 and August 1949 reveals what may be considered an approximation to of the yearly mean total eddy transport of sensible heat. There exists a maximum transport across 55°N. with a marked and continuous divergence of the transport south of 55°N. and a marked and continuous convergence of the transport north of this latitude.

The total eddy transport of latent heat, Table I, differs slightly from winter to summer. During the winter time the total eddy transport of latent heat exhibits the double maximum at latitudes 35° and 55°N. and the transport decreases sharply to the north of 65°N. Contrasted with these conditions, the total eddy transport during the summer has a slight single maximum at 45°N. but is relatively constant between 35°N. and 75°N. Although the transport in the summer time is generally smaller in middle latitudes than in the same region during the winter, it is much larger at higher latitudes. An additional feature is the southward transport of latent heat across 25°N. in the summer time as contrasted with a winter-time northward transport across the same latitude.

When one combines the transport of energy in latent form and as sensible heat from the data for February and August 1949 a picture of the total eddy energy-transport as the sum of these energy forms is obtained. These figures should approximate the terms $\int\int (c_p T + ql)(\rho v)' dx dz$ of Eq. (2). This combined total transport, Table II, last column, shows a maximum at 55°N. decreasing both south and north of this latitude. A southward transport exists across 25°N.

TABLE II. The total eddy transport of sensible + latent heat, integrated vertically to the top of the atmosphere, transformed into the total transport across a latitude circle under the assumption that the data for North America are representative for the entire hemisphere. Values given for various latitudes and times of year. Units in cal/sec $\times 10^{11}$. F = Feb., A = Aug.

Lat. °N.	F0300	F1500	A1500	F0300 + F1500	F1500 + A1500
25	+ 1473·8	− 1540·9	− 2384·9	− 51·7	− 1987·5
35	+ 8208·2	+ 6684·7	+ 1230·0	+ 7432·5	+ 3975·4
45	+ 5335·6	+ 6909·4	+ 4760·1	+ 6122·5	+ 6182·2
55	+ 24582·0	+ 12550·3	+ 5585·3	+ 18656·6	+ 9170·3
65	+ 3888·7	+ 6014·8	+ 2991·3	+ 5948·8	+ 4710·1
75	+ 889·2	+ 1064·9	+ 2387·3	+ 775·4	+ 1673·9

In a rough manner it is possible to make a comparison of the magnitudes of the total eddy transport of energy as the sum of sensible and latent heat with published values for the total energy transport required by the radiation balance indicating that horizontal eddy-transport mechanisms accomplish a major portion of the required energy transport. Preliminary hemispherical determinations of long-term means of all the quantities entering into the global energy-balance and the significance of the eddy-energy flux in the total scheme have been completed by the author and will be reported in a forthcoming paper. However, in view of the fact that only a small portion of the hemisphere has been sampled for a short time-period in the investigation reported here, the magnitudes should not be considered as approaching true means. On the other hand, considerable weight must be attached to the information which such data give us regarding the variation of the eddy-transport processes in both time and space.

It is of great interest to determine what portion of the eddy transport of sensible heat is accomplished in the lower half of the atmosphere below 500 mb. It is also of interest to determine the difference in the magnitude of the transport of sensible heat at various latitudes from summer to winter. The analysis was carried out for the mid-latitude regions 35°N. to 65°N. The percentages of the total transport occurring below 500 mb based upon the data for February 1949 (0300 GMT + 1500 GMT) are as follows :

35°N.	45°N.	55°N.	65°N.
62	88	68	73

It can be seen that there is a slight variation in the percentages but not greater than might be expected from the nature of the data.

In comparing the rate of transport during the winter with the mean yearly transport, it is found that the ratio of the transport as given by the February data to the mean transport as given by the February and August data is :

35°N.	45°N.	55°N.	65°N.
1·93	1·19	1·42	1·34

It was also possible with the data at hand to show more clearly the relative magnitudes of the local eddy flux of sensible heat and the standing eddy flux. For this purpose a selection of five stations along latitude 45°N. was made. These included Spokane, Wash. ; Tatoosh, Wash. ; Bismark, N.D. ; Int. Falls, Minn. ; and Caribou, Maine.

The local eddy flux was computed for February 1949 for each of these stations in a manner similar to Priestley's. The standing eddy flux was then obtained from a consideration of the space correlation of the time means for each of these stations. The mean local eddy flux and the standing eddy flux for these stations are given in Table III. It can be seen that both terms are of the same order of magnitude.

TABLE III. A comparison of the magnitude of the mean local eddy flux and standing eddy flux of sensible heat at the various levels in the atmosphere over regions of the United States at latitude 45°N. Units in °A × m/sec per cm of longitude for a vertical distance equal to one mb of pressure difference.

mb	Mean local eddy flux	Standing eddy flux
850	+ 10·02	+ 14·41
700	+ 10·29	+ 7·43
500	+ 11·39	+ 2·52
300	+ 1·19	− 1·25
200	− 12·08	− 12·96
100	+ 1·34	− 0·27

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Physical interactions between tropical and temperate latitudes

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SUMMARY

The poleward transports of momentum and angular momentum, enthalpy, water vapour, and total energy are studied at a number of upper-air stations in the high-pressure belt. The results confirm earlier suggestions of a mean meridional circulation across this belt ; this circulation is found to contribute about one half the total poleward transport of both energy and angular momentum, and also to make a substantial contribution to the maintenance of the total kinetic energy of the atmosphere. The meridional circulation appears notably stronger in winter than in summer. The poleward transport of angular momentum to the Westerlies reflects this variation, whereas there is relatively little seasonal variation in the contribution from eddy flux (horizontal rotations) or in the total generation of angular momentum in the Easterlies of low latitudes.

Variations of eddy flux with height are examined and certain general features (Priestley, *Q.J.*, 1949) confirmed. The lowest 500 mb of atmosphere make no appreciable contribution to the net eddy flow of momentum, the transport occurring mainly at the 300 mb and 200 mb levels.

The results also provide a measure of the difference between total annual precipitation and evaporation in the tropical zone.

1. The problem and methods

(*a*) *Introduction*

The mean pattern of surface winds reveals a low-latitude zone where the zonal component is from the E. and a middle-latitude zone where it is from the W. Since the sole force external to the atmosphere is the surface friction, this implies the existence of two main zones, separated by the high-pressure region of latitudes 30° - 35°, where the average rates of generation of angular momentum are opposite and approximately equal. Considerations of radiation balance by various workers have led to the conclusion that the earth and atmosphere, as a whole, receive a net radiation-surplus in low latitudes, but polewards of about 35° the outgoing exceeds the incoming radiation. These facts are well known and the coincidence provides evidence, if such were needed, of the intimate connection between thermal and mechanical processes in the atmosphere.

Of recent years much interest has centred in the mechanical processes in the layers around the tropopause. This work has led to the identification of a narrow belt of maximum westerly winds whose mean position, about latitude 35°, separates a circumpolar vortex from an equatorial zone ; and to the theory that these two upper zones are governed respectively by the quite different conditions of constant vorticity and constant vorticity-transport.

The angular-momentum balance, the energy balance, and the upper-wind distribution constitute three avenues of attack on the general-circulation problem which converge on one point ; namely, that there exist two principal zones of the atmosphere whose latitude of separation is about 30° - 35°. We shall speak of these zones as ' tropical ' and ' temperate,' disregarding the more normal (astrono-

mical) definition of these terms which has little physical significance in the problems discussed here. Namias (1947) has studied the simultaneous variations of surface-wind strength in the two zones, and thence developed a promising method of extended forecasting. To support this practical work and allow its extension in other directions there is a need for more fundamental study of the interactions between the two principal zones in terms of the interchange of energy and momentum. The special importance of processes occurring in latitudes 30° to 35° is underlined, for the reasons above and the consequence that across these latitudes the total poleward transports of both energy and angular momentum must attain their maximum values. These interactions are here to be examined.

It is shown by Namias that at times the pattern differs greatly from its normal, to the extent of the disappearance or even partial reversal of the principal zones. At all times there are still large variations from the pattern of zonal symmetry. This latter fact is implicit in the classical arguments concerning the transport by horizontal rotations in the maintenance of the global wind and energy distribution. Nevertheless across latitudes 30° - 35° it has been shown that the eddy-transport of momentum, and will be shown that the eddy-transport of energy, are, on the best evidence available, insufficient for long-period interzonal balance. In the former case a mean meridional circulation of order 1 to 2 m/sec equatorwards at low, and polewards at tropopause, levels would make good the deficiency (Priestley 1950). Whether this is also true of the energy balance is to be discussed below, together with other consequences of such a circulation, for whose existence there is much consistent evidence.

(*b*) *Method of attack*

The intention and method of studying physical exchanges between latitudes were formulated simultaneously by Bjerknes (1948) and the present writer (1948), and already some results have appeared (Priestley 1949, 1950 ; Widger 1949). While the objectives appear to be identical the methods differ sufficiently to merit some discussion here.

Though Widger's results were presented in statistical form, the method described by him and by Bjerknes is essentially synoptic, being based on daily series of hemispheric charts from which the geostrophic wind is read off at a sufficient number of points to allow the angular-momentum transport to be assessed. It admits of ready extension to the assessment of energy transport. The method is extensive, but the impossibility of objective drawing of accurate hemispheric charts at 300 mb and above makes it at present inapplicable to just those levels where the principal transport may occur. Moreover the use of the geostrophic assumption is fundamentally restrictive for the mean geostrophic velocity, taken right round a circle of latitude in an isobaric surface, and the mean geostrophic mass flow in a constant-level surface are zero. In other words, the method eliminates any mean meridional circulation and the contribution which this may make to the transports. Quite apart from this, any results from the geostrophic assumption in low latitudes are suspect.

The statistical method developed by the writer employs the values of wind, temperature and pressure as observed at stations making regular ascents, and is not

open to any of the above objections, besides lending itself more readily to studies covering long periods of time. But it cannot be applied to areas where soundings with daily regularity are not available, and even to extend it to all sounding bases would involve enormous labour. Unless this task is faced there must always remain some doubt as to whether the stations selected are truly representative ; while the ' standing-eddy flux,' due to horizontal rotations in the semi-permanent pressure systems, must either be ignored or assessed independently from mean charts by the geostrophic method.

There is reason and evidence to suppose that the meridional circulation is highly variable, both in space and time, and is thus an essentially synoptic phenomenon. Nevertheless the problem of meridional circulation may, in the first instance, be attacked by the statistical method. Indeed no other course is open since the geostrophic method suppresses any such component and the close world-wide network of precise observations, by which alone the synoptic approach could be freed from restrictive assumptions, is not yet available. Locally and temporarily the mean meridional circulation is hidden by the much larger overturnings due to temperature contrasts between longitudes, and its existence can be established only by analysis of large amounts of material. Furthermore the energy and momentum are known to be self-adjusting within periods measured in days ; and the evidence, which follows, that the extended troughs and anticyclones (eddy flux) are inadequate channels for full replenishment *in the mean* therefore implies that even for short-period studies the synoptic approach will remain of limited usefulness until it can detect the instantaneous value of the net meridional mass interchange, level by level.

In general, then, neither method is completely independent of the other, and choice must be determined by the nature of the problem under consideration. In the present case we are concerned with the annual and seasonal transports across a particular narrow belt of latitude, and since it cannot be argued *a priori* that the meridional circulation has no existence or importance, the method using direct observational material is to be preferred.

(*c*) *Requirements for balance*

There are certain conditions, in the nature of balance requirements, which the total annual flow of angular momentum, heat, *etc.*, across any latitude must fulfil. These conditions have been described in the literature and need be further discussed here only in so far as it is necessary to give quantitative expression to the requirements of the flux across the particular zone under consideration.

The radiation balance has been studied by a number of workers, perhaps most thoroughly by Baur and Phillips (1935), from whose figures we infer a mean poleward transport of heat across latitude 35° of 2×10^{7} cal cm^{-1} min^{-1} as the requirement for annual balance.

The quantitative requirements for momentum balance are not available in the literature and must be assessed *ab initio*. The writer (1950) has estimated the total annual generation of angular momentum between the equator and latitude 30° or 35°S. at $1{\cdot}3\times10^{34}$ g cm^{2}/sec. This, and an approximately equal amount

generated in the northern tropical zone, would require to flow polewards across the bounding latitude to its ' sink ' region at the surface in the westerlies of middle latitudes. The first estimate was, however, rough and a somewhat more refined one has been made for the present work, from the same oceanic wind data as previously used. The frictional stress at a given time and place is represented by

$$\tau = \kappa \rho v^2$$

in direction opposed to that of surface wind v. In the earlier computation the annual vector-mean wind was determined for each 5° latitude zone, and this was used directly in the above expression to give an approximation to $\bar{\tau}_x$, the mean eastward component of stress. On the present occasion each wind-rose (applying to one month and a 5° latitude-longitude area) was treated individually and the mean τ_x extracted by considering the frequency and mean strength of winds in each octant : the results were then averaged over all longitudes and over the four months of January, April, July and October. The analysis was extended to the northern latitudes 0° - 30° also*. The method as described does not allow for the variability of wind speed from a given direction ; the correction for this was found, by reference to a smaller but representative number of more detailed wind roses, to be fairly uniform, averaging 20 per cent. for the latitudes in question. As a result of this survey, embodying the above correction, the total torque of the surface easterlies was assessed at $1{\cdot}9 \times 10^{34}$ g cm^2/sec per year for each hemisphere.

On the other hand the, assumed constant, value of 0·0025 for κ adopted previously may be if anything too high. In a recent review, Deacon (1950) states that the mean value of all determinations in light and moderate winds is 0·0013 ; this further correction would bring the total annual generation of angular momentum in both hemispheres combined to $2{\cdot}0 \times 10^{34}$ g cm^2/sec. This figure will be adopted here. It is more likely to be an under- than an over-estimate owing to the underweighting of strong wind occasions, the enhanced effect of friction over land, the neglect of the torque due to pressure differences across mountain barriers, and the inclusion in the averaging of κ of some determinations made over relatively confined waters.

On the average, then, heat must be carried polewards across each of the bounding zones in latitudes 30° - 35° at the rate of 2×10^7 cal cm^{-1} min^{-1}, and angular momentum at $1{\cdot}0 \times 10^{34}$ g cm^2 sec^{-1} per year, equivalent to a stress of $1{\cdot}8 \times 10^8$ dynes cm^{-1}. Both of these figures are necessarily approximate, but they are the best available, the second being almost certainly on the conservative side. The degree to which these transports are achieved by the horizontal-eddy mechanism becomes a criterion for the adequacy of the latter and for the existence of some other mechanism of transport.

Ideally, a similar criterion would apply in the problem of the water balance, which requires that the transport of atmospheric water across latitudes 35° shall equal the difference between precipitation and evaporation from the surface within these boundaries. These latter amounts have been variously estimated, most recently by Reichel (1949), whose figures imply an annual excess of evaporation for

* This work has involved the analysis of about 3,000 surface-wind frequency roses covering the oceans between 30°S. and 55°N. for each of the four months. It is hoped to present and discuss the full results elsewhere. Those given here and in section 3(d) are brought forward as being of special relevance to the subject of this paper.

the whole $\pm 35°$ region amounting to $3{\cdot}7 \times 10^{19}$ cm^3 of water per year, an average of 130 mm over the area. Without discounting the value of such work, it must be borne in mind that Reichel's or any comparable figure depends on a small residual between two quantities of which one (evaporation) cannot at present be directly measured while the other (precipitation) is inferred from island records. In the circumstances the available figures for precipitation-evaporation differences can provide no useful criterion for the adequacy of eddy transports. Rather is the position reversed, and if the whole atmospheric-water transport across the bounding latitudes can be assessed, as will here be attempted, the value could be used to test the accuracy of Reichel's and other estimates of precipitation-evaporation differences. With sufficient accuracy we may neglect the atmospheric transport of water in liquid form as compared with that carried as vapour, equating the latter to the total transport.

2. Data and results

Stations with daily regularity of soundings up to 150 mb are not plentiful, but nevertheless sufficient to enforce some selection of material for the first assay. The selection was made to be as representative as possible of the latitude zone as a whole. Between 30° and 35°, in the two hemispheres together, the oceans occupy about two-thirds of the area while most of the land is of the arid or desert type.

The material here brought under study consists of daily, or more frequent, upper-air soundings of temperature, pressure, humidity, and wind from the following places and periods :

Gibraltar (36° 09′ N., 05° 21′ W.). Jan. 1948 — Dec. 1948 incl.

Habbaniyah (33° 22′ N., 43° 34′ E.). Jan. 1948 — Dec. 1949 incl., referred to as years 1 and 2.

Norfolk Island (29° 03′ S., 167° 56′ E.). July 1944 — June 1945 incl.

Auckland (36° 51′ S., 174° 46′ E.). Jan. 1948 — Feb. 1950, excluding Jan. and Feb. 1949 which were omitted as there were several days in each month when soundings were either not made or failed to reach sufficient height. For Auckland, 1948 will be referred to as Year 1, Mar. 1949 — Feb. 1950 as Year 2 in the tables and discussion.

These six place-years, with two from an arid land region, three oceanic and one (Gibraltar) as intermediate, are as representative as could be made. Material from the North American continent has been deliberately excluded, since this is already under complementary analysis by workers in the U.S.A. ; but also because that region, more than those considered here, must come under suspicion when representativeness for the globe as a whole is required. The Rocky Mountain barrier is thought to exert a marked local influence on the nature and intensity of day-to-day meridional exchange, while mean upper-air charts clearly demonstrate the local disturbance which appears in the mean zonal-flow pattern in the North American region.

Although differences between the southern and northern hemispheres undoubtedly exist, the data will be regarded as a single group of material, to a first approximation homogeneous. It would be possible to adduce hemispheric differences, but it is preferred to leave such refinement until far more data have come under review.

The method of analysis of the data, with the terminology and notation, follow ose of the earlier (1949) work, with two exceptions : following White (1950), the ix there called ' sensible heat ' is more properly referred to as that of ' enthalpy,' hile in the latent-heat flux the humidity mixing ratio x is replaced by specific midity q for formal correctness (this entails no significant amendment to the results ported earlier). Zonal wind components U will be regarded as positive when esterly, and the meridional component V and the corresponding flux as positive hen directed poleward.

The annual values of eddy flux at chosen pressure levels and the total (vertically tegrated) values are presented in Table I (enthalpy), Table II (heat flux in latent rm), and Table III (momentum). Table IV shows the annual mean temperature, midity, zonal and meridional wind component averaged over all the six place-ears.

Tables I to III constitute averages, for the whole year, station by station. In ables V to VII the evaluations of eddy flux are presented as averages for all stations ason by season.

TABLE I. Annual eddy flux of enthalpy

Units : 10^2 °A cm sec^{-1} in upper portion, 10^7 cal cm^{-1} min^{-1} in bottom row.

$\overline{V'T'}$:	Gibraltar	Habbaniyah Year 1	Habbaniyah Year 2	Norfolk Is.	Auckland Year 2*	Mean
0 mb	−0·2	+4·7	+3·5	+9·2	+7·0	+4·8
0	+1·7	−0·2	+1·3	+4·2	+6·4	+2·6
0	−0·9	−0·5	−0·1	+2·7	+9·7	+2·2
0	−2·1	−1·2	+0·7	+4·4	+8·4	+2·1
0	−2·9	−4·0	+7·0		+6·0	+1·5
0	+5·1	+2·8	+7·0		+7·5	+5·6
0	+8·2	+4·6	+1·2		+2·0	+4·0
otal eddy flux	+0·10	+0·08	+0·41		+0·90	+0·39

TABLE II. Annual eddy flux of heat in latent form

Units : 10^2 °A cm sec^{-1} in upper portion, 10^7 cal cm^{-1} min^{-1} in bottom row.

$\overline{V'q'}$:	Gibraltar	Habbaniyah Year 1	Habbaniyah Year 2	Norfolk Is.	Auckland Year 2*	Mean
0 mb	+2·4	+3·7	+3·6	+14·3	+11·8	+7·1
0	+6·6	+3·0	+4·8	+11·5	+4·3	+6·0
0	+1·6	+1·1	+1·9	+4·4	+3·2	+2·5
0	+0·1	+0·1	+0·7	+1·9	+2·3	+1·0
0	−0·1	+0·1	+0·2		+0·3	+0·1
0						
0						
otal eddy flux	+0·30	+0·22	+0·29	+0·83	+0·56	+0·44

* In Year 1 (Auckland), temperature and humidity soundings were not sufficiently regular for the analysis.

TABLE III. Annual eddy flux of momentum
Units : 10^5 cm^2 sec^{-2} in upper portion ; 10^5 dyne cm^{-1} in bottom row.

$\overline{V'U'}$ at :	Gibraltar	Habbaniyah Year 1	Habbaniyah Year 2	Norfolk Is.	Auckland Year 1	Auckland Year 2	Mean
900 mb	−0·5	−0·8	−1·0	−0·5	−1·1	−0·9	−0·8
700	+0·3	−0·1	+0·0	−0·2	+0·2	−0·0	+0·0
500	+1·1	+1·4	+0·9	+0·0	+0·8	+1·5	+1·0
400	+1·7	+2·2	+2·0	−0·2	+1·4	+3·6	+1·8
300	+2·6	+3·7	+3·0		+2·2	+6·0	+3·5
200	+4·6	+3·5	+2·7			+7·2	+4·5
150	+2·1	+0·7	+1·4			+4·2	+2·1
Integrated Reynolds's stress	+1·1	+1·0	+0·8		(+0·7)	+1·9	+1·1

TABLE IV. Mean values of the elements

p (mb)	$\bar{T}$ (°A)	$\bar{q}$ (gm/Kg)	$g\bar{z}$ (cm^2/sec^2) $\times 10^8$	$c_p\bar{T}+L\bar{q}+Ag\bar{z}$ (cal/g)	$\bar{U}$ (cm/sec)	$\bar{V}$ (cm/sec)	$\bar{V}_1$ (cm/sec)
0							
50	(215·0)	0	(20·73)	(101·1)	(250)		+ 14·5
100	(215·0)	0	(16·10)	(90·0)	(1200)		+ 59·5
150	215·0	0	13·53	84·1	2200	+ 210	+ 149·5
200	219·5	0	11·71	80·7	2430	+ 241	+ 180·5
250	225·5	0	10·28	78·7	2300		+ 150·5
300	232·5	0·05	9·08	77·5	1900	+ 159	+ 98·5
350	240·2	0·2	8·03	76·9	1650		+ 44·5
400	246·9	0·3	7·09	76·5	1430	+ 72	+ 11·5
450	253·1	0·5	6·25	75·9	1250		+ 11.5
500	258·5	0·7	5·48	75·5	1080	+ 72	+ 11·5
550	263·1	0·9	4·77	75·1	950		+ 2·5
600	267·5	1·2	4·11	74·7	850		− 10·5
650	271·5	1·6	3·49	74·5	730		− 25·5
700	275·5	2·1	2·91	74·3	620	+ 13	− 47·5
750	279·0	2·7	2·36	74·2	530		− 82·5
800	282·0	3·4	1·84	74·1	430		−112·5
850	284·7	4·3	1·35	74·0	330		−135·5
900	286·9	5·4	0·88	74·2	230	− 88	−148·5
950	(289·0)	(6·7)	(0·43)	(74·4)	(150)		−133·5
1000	(291·2)	(8·3)	0	(74·8)	(0)		− 80

( ) denotes estimated value. The total toroidal flux of energy is not greatly affected by the uncertainties at these level

TABLE V. Seasonal eddy flux of enthalpy
Units : 10^2 °A cm sec^{-1} in upper portion ; 10^7 cal cm^{-1} min^{-1} in bottom row.

$\overline{V'T'}$ at :	N.hem. S.hem.	Jan.-Feb. Jul.-Aug.	Mar.-Apr. Sep.-Oct.	May-Jun. Nov.-Dec.	Jul.-Aug. Jan.-Feb.	Sep.-Oct. Mar.-Apr.	Nov.-Dec. May-Jun.	Mea
900 mb		+ 9·0	+ 5·9	− 0·3	+ 2·1	+ 5·4	+ 6·4	+ 4·8
700		+ 3·5	+ 3·0	− 2·0	+ 2·1	+ 4·1	+ 5·2	+ 2·6
500		+ 6·8	+ 2·1	− 5·1	− 0·5	+ 4·3	+ 5·5	+ 2·2
400		+ 2·3	+ 2·7	− 2·2	− 1·2	+ 6·0	+ 4·8	+ 2·7
300		+ 6·6	+ 3·6	− 6·8	+ 2·1	+ 2·1	+ 1·5	+ 1·5
200		+ 6·5	+ 13·0	+ 2·7	+ 5·0	+ 3·2	+ 3·1	+ 5·6
150		+ 4·1	+ 9·4	+ 2·5	− 1·3	+ 6·5	+ 2·3	+ 4·0
Total eddy flux		+ 0·77	+ 0·64	− 0·27	+ 0·17	+ 0·56	+ 0·59	+ 0·3

TABLE VI. Seasonal eddy flux of heat in latent form
Units : 10^2 °A cm sec^{-1} in upper portion ; 10^7 cal cm^{-1} min^{-1} in bottom row.

$\overline{V'q'}$:	N.hem. Jan.-Feb. S.hem. Jul.-Aug.	Mar.-Apr. Sep.-Oct.	May-Jun. Nov.-Dec.	Jul.-Aug. Jan.-Feb.	Sep.-Oct. Mar.-Apr.	Nov.-Dec. May-Jun.	Mean
mb	+ 9·0	+ 5·5	+ 4·7	+ 6·3	+ 7·6	+ 9·7	+ 7·1
	+ 11·7	+ 4·7	+ 1·3	+ 4·6	+ 7·2	+ 6·8	+ 6·0
	+ 2·7	+ 2·3	+ 2·2	+ 2·1	+ 2·8	+ 2·7	+ 2·5
	+ 1·1	+ 0·3	+ 1·2	+ 1·1	+ 1·4	+ 0·9	+ 1·0
	0	0	+ 0·1	+ 0·1	+ 0·2	+ 0·1	+ 0·1
tal eddy flux	+ 0·66	+ 0·35	+ 0·25	+ 0·36	+ 0·50	+ 0·53	+ 0·44

TABLE VII. Seasonal eddy flux of momentum
Units: 10^5 cm^2 sec^{-2} in upper portion ; 10^8 dyne cm^{-1} in bottom row.

$\overline{U'}$:	N.hem. Jan.-Feb. S.hem. Jul.-Aug.	Mar.-Apr. Sep.-Oct.	May-Jun. Nov.-Dec.	Jul.-Aug. Jan.-Feb.	Sep.-Oct. Mar.-Apr.	Nov.-Dec. May-Jun.	Mean
	− 0·5	− 1·2	− 0·3	− 0·6	− 1·1	− 1·1	− 0·8
	+ 0·3	+ 0·1	+ 0·2	− 0·1	+ 0·2	− 0·4	+ 0·0
	+ 1·4	+ 0·7	+ 1·2	+ 1·0	+ 0·7	+ 0·8	+ 1·0
	+ 2·2	+ 2·0	+ 1·4	+ 1·9	+ 1·7	+ 1·3	+ 1·8
	+ 6·0	+ 2·9	+ 2·9	+ 4·2	+ 2·9	+ 2·2	+ 3·5
	+ 7·8	+ 3·5	+ 2·5	+ 6·3	+ 3·8	+ 3·0	+ 4·5
	+ 2·1	+ 0·1	+ 1·3	+ 4·1	+ 3·4	+ 1·1	+ 2·1
tegrated eynolds's ress	+ 1·8	+ 0·7	+ 0·9	+ 1·6	+ 1·1	+ 0·6	+ 1·1

3. Interpretation and discussion

(*a*) *The adequacy of the eddy transport*

The first and most important comparison can now be made. The mean eddy lux of momentum, 1·1 × 10^8 dynes/cm, is equivalent to a poleward flow of angular nomentum totalling 6 × 10^{33} g cm^2 sec^{-1} per year, about one-half of what has been stimated as the balance requirement. In each case the eddy transport was in the ight sense and (with the possible exception of Norfolk Island for which the higher-evel data were missing) amounted to an appreciable fraction of the requirement.

Turning to the eddy flux of heat quantities we again find a mean figure, ·83 × 10^7 cal cm^{-1} min^{-1}, approximately half that estimated for balance. No ndividual total attains the requirement figure but again each of the individuals nakes a substantial contribution in the proper sense of a poleward heat flow. For he mean of a winter and a summer month in North America, White (1950) finds comparable figure of 0·73 × 10^7 in latitude 35°, also divided approximately equally etween the enthalpy and latent-heat components.

The flux in latent form of 0·44 × 10^7 cal cm^{-1} min^{-1} is equivalent to a yearly ow of 1·3 × 10^{19} grams of atmospheric water poleward across latitude 33°. This s in fair agreement with Reichel's estimate of precipitation-evaporation differences.

These comparisons relate only to the local values of eddy flux. The writer as found that the standing-eddy flux of momentum is only a minor factor, but has

made no estimate of this component in the case of the heat quantities. For th North American continent White (1950) finds the standing-eddy flux of enthalp to be of the same sign and comparable with, though smaller than, the local edd flux. In view of the fact that this region is the site of the most marked distortio in the generally zonal pattern of upper winds, it is highly probable that the standing eddy flux for the globe as a whole is less than that for the North American regio and therefore unlikely to account for a major part of the discrepancy between edd transport and the balance requirement.

In summary the results indicate that horizontal rotations are important age for the poleward transport of energy and momentum across these latitudes, bu leave approximately a half of these transports unaccounted for.

(*b*) *The meridional circulation as a transport agent*

Within the last year argument and evidence, of varied nature, have been pu forward for the existence of a mean meridional circulation between the two prin cipal zones (Brewer 1949, Riehl 1950, Priestley 1950, Riehl and Yeh 1950). Th last two papers agree in establishing a $\bar{V}$ of -100 cm/sec at surface levels, and whil there is some disparity in the various values of $\bar{V}$ assigned to other levels they ar not essentially at variance, as will be shown below. The mean values of $\bar{V}$ in Table I may not prove completely representative, but they are the only direct assessmen which have been made so far and are therefore to be adopted as a basis for estimatin the flux of momentum and energy effected by the mean meridional circulatio between the zones. At the present stage it would be improper to insist on an exact magnitude or particular form for $\bar{V}$ *qua* function of p ; but since we are her concerned purely with the transport by the mean meridional circulation, we mus be careful to exclude any contribution from the mean horizontal rotations (standing eddy flux), especially since the latter cannot be properly assessed by the presen method (see section 1 (b)).

The values of $\bar{V}$ are accordingly adjusted to conform to zero-pressure-mea by subtracting a constant value from the figures while retaining the condition that $\bar{V}$ tends to zero with p and is about -100 cm/sec at 1,000 mb. The values s derived are entered as $\bar{V}_1$, in Table IV.

The mean toroidal flux of momentum, written in the usual notation

$$\frac{1}{g}\int_0^{p_0} \bar{V}_1 \bar{U} dp,$$

can now be found directly from Table IV as $0{\cdot}65 \times 10^8$ dynes/cm, the equivalen of 4×10^{33} g cm^2 sec^{-1} per year of angular momentum, of the same order as th eddy flux and in the same sense. This is computed on the assumption that $\bar{V}_1$ uniform throughout the year ; with seasonal variations in the sense expected (sectio 3(d)) its contribution to the annual momentum transport will be larger.

The corresponding expression for toroidal energy flux comprises three term the fluxes of enthalpy, latent heat, and kinetic energy. As in the case of eddy flu the contribution from the last is negligible by comparison with the others, and wi

neglected. When there exists a mean meridional circulation, then the energy be transferred between zones does not equal the net radiation-energy surplus of the tropical zone, but account must also be taken of the mean rate of working against the external forces (gravity) in the tropical zone, and the equation of lance becomes

$$R - W = \text{eddy flux of energy} + \oint ds \int_{z=0}^{\infty} \rho(c_p\overline{T} + L\overline{q})\,\overline{V}_1 dz$$

here ds denotes an element of arc of the latitude circle. If dS be used to denote horizontal element of area in the tropical zone across which the mean upwards locity is w, then by continuity

$$\int_{dS} \rho w dS = -\oint ds \int_0^z \rho \overline{V}_1 dz$$

'e shall write, for brevity, $\int_0^z \rho\overline{V}_1 dz = f(z)$, whence the expression for the total te of working against and by gravity in the tropical and temperate zones respec-vely is

$$W = \int_{z=0}^{\infty}\int_{dS} g\rho w dS dz = -g\oint ds \int_0^{\infty} f(z)dz$$

$$= -g\oint ds\left\{\left[zf(z)\right]_0^{\infty} - \int_0^{\infty} zf'(z)dz\right\} \text{ by parts. Now as}$$

tends to infinity, the finiteness of W requires that $f(z)$ behaves like $z^{-1-\delta}$, where is a positive number, so that

$$\left[zf(z)\right]_0^{\infty} = 0$$

hence

$$W = \oint ds \int_0^{\infty} g\rho z\overline{V}_1 dz$$

nd the equation of balance may be written, by transposition,

$$R = \text{eddy flux of energy} + \frac{1}{g}\oint ds \int_0^{p_0} (c_pT + Lq + Ag\overline{z})\,\overline{V}_1 dp$$

here A is the reciprocal of the mechanical equivalent of heat.

Other workers have proceeded direct to this equation, identifying the right and side as the total energy flux and the last term in it as the flux of potential nergy ; but the above derivation may be preferred by those who find difficulty n appreciating, immediately, the physical meaning of a ' transport of positional nergy,' or who require a method of formulation applicable to problems involving ne energy transport across boundaries which are not everywhere vertical

The value of the last integral, per unit length ds, is found from Table IV to e $+ 1{\cdot}6 \times 10^7$ cal cm^{-1} min^{-1}. It is of interest to note that this is made up of ontributions of $-3{\cdot}2$, $-0{\cdot}6$, and $+5{\cdot}4 \times 10^7$ from the three terms in the order hey appear above, so that W substantially exceeds R in magnitude. We compare $+2{\cdot}0 \times 10^7$ as the balance requirement for R and $+0{\cdot}83 \times 10^7$ as the observed ddy flux.

In these toroidal-flux evaluations the value of p_0 employed was 1,000 m
Allowance for a true mean surface pressure somewhat greater than this would ha
complicated the working without changing the results appreciably. A more serio
difficulty is the ignorance of the proper values of $\bar{V}$ and $\bar{V}_1$ at levels above 150 m
to which the observations do not regularly extend : those adopted are such th
$\bar{V}_1$ tends to zero with p, but there must remain the possibility of having overlooke
an important contribution to the energy balance arising from even very slow ci
culations at these high levels. In particular, a reverse circulation above 150 m
would reduce the W term, so bringing the total nearer to the estimated balanc
requirement. Without pressing approximate considerations too far, the evidenc
may be summed as indicating strongly that agreement between flux and balanc
requirement is satisfactory if, and only if, the toroidal contribution is included
this applies to both energy and momentum.

The figures for $\bar{V}_1$ and $\dot{q}$ in Table IV imply a mass flow of atmospheric wate
towards the equator of amount $1{\cdot}8 \times 10^{19}$ g per year. Adding this algebraicall
to the poleward eddy flow of $1{\cdot}3 \times 10^{19}$, we infer a net equatorward flow of 5×10^{1}
g per year, or an average excess of precipitation over evaporation in the tropica
zone of between 1 and 2 in. It is thought that the sense and magnitude of thi
difference lies within the accuracy with which it has been and can at present b
otherwise assessed for this largely oceanic region.

(c) *Further discussion of the mean meridional circulation*

From the calculations just made it becomes clear that the potential importanc
of the mean meridional circulation must be considered in all global problems in
volving transport of energy, momentum, water vapour, *etc.* Ignorance as to it
magnitude, over short periods and in the average over longer ones, will set a corres
ponding limit to our understanding of the larger atmospheric pulsations, such a
changes in weather type or in the zonal wind indices.

It is desirable then to assess as accurately as possible, weighing all evidenc
and argument, the mean rate of the circulation. The requirements of balanc
support the belief that the figures for $\bar{V}_1$ in Table IV represent correctly the mai
features of the circulation. The principal features, all explicit or implicit in thi
single column of drift speeds are : an equatorward mean drift in the lowest layer
with counterdrift at near-tropopause levels, each having components of orde
150 cm/sec and a total mass flow of $1{\cdot}1 \times 10^{19}$ g per day ; air ascending in th
tropical zone at a mean rate of 130 m per day at 500 mb and of 120 m per day throug
the tropopause (approx. 200 mb) level, with similar mean rates of descent to pole
ward. The figures are quoted specifically, but must of course be viewed broadly
and on this basis may be compared with estimates made elsewhere from differen
evidence.

The present author (1950) and Riehl and Yeh (1950) have demonstrated th
reality of the mean equatorward drift at about 100 cm/sec at surface levels. Th
last-named, with the professed intention of assessing the lower limit to the circu
lation, have assumed that this drift falls to zero at 1 km and is reversed above tha
level. In consequence their estimates of *minimal* mean vertical motion, *etc.*, fal

hort of the figures proposed here. But there appears no sound reason for supposing he mean surface drift across latitudes 30° — 35° to be an effect of surface friction : ather does the evidence from this and other latitudes discountenance this assumption, and it is reasonable to suppose the drift to be continued through a much greater epth of atmosphere.

Brewer (1949) sets the mean rate of descent through the temperate tropopause t 50 m, with an upper limit at 100 m per day. While these are not greatly at variance ith the 120 m per day now put forward one would, following Brewer, be inclined o avoid the stress placed here on the importance of the circulation in inter-latitude ux processes. In Brewer's workings, however, the main quantitative evidence elates not to the mean vertical velocity w itself but to the ratio w/K, where K is he coefficient of eddy diffusion, a quantity whose exact value is more uncertain han that of w. Brewer has used a constant value of K for a layer where Lettau 1947) has shown it to experience a most marked increase with height ; moreover w of 120 m per day would imply, according to the theory, a K much nearer to that ound by Lettau for the 10 — 13 km layer than that which Brewer himself assumes. n view of these points a mean rate of descent of 120 m per day through the temperate tropopause is quite consistent with Brewer's explanation of the humidity listribution of the lower stratosphere.

The circulation speeds presented here, therefore, do not conflict with previous evidence at any point, and we may employ them to estimate the energy released by air in its meridional circulation, multiplying the rate of mass flow by the energy released in an *average* circuit. This average circuit can fairly be represented by equatorward and poleward motion at 850 mb and 220 mb respectively (these being the levels below and above which one half the horizontal mass flow occurs), the ascent and descent taking place in the latitudes which bisect the zonal areas respectively equatorward and poleward of 35°. For practical purposes these latitudes may be taken as 15° and 50° respectively ; average temperatures for certain relevant pressure levels and latitudes are set out in Table VIII.

TABLE VIII. Mean temperatures (°F) for relevant pressures and latitudes

Latitude	Pressure (mb)			
	850	750	500	200
0°	66	56	22	−48
15°	62	54	20	−47
25°	59	51	16	−51
50°	34	26	−8	−66

One gram of air describing the circuit between latitudes 15° and 50° and pressures 850 and 220 mb in the sense indicated would liberate $5{\cdot}6 \times 10^7$ ergs ; multiplying by $1{\cdot}1 \times 10^{19}$ g per day we infer a total release of energy by the mean meridional circulation at the rate of 6×10^{26} ergs per day. This is the same figure as given by Brewer, though the component factors are quite different. Brunt (1926) has estimated the mean rate of dissipation of kinetic energy for the whole atmosphere as 2×10^{27} ergs per day, and the comparison serves to substantiate Brewer's con-

clusion that the mean circulation makes an important contribution to the maintenanc of the total kinetic energy of the atmosphere. Such a conclusion is wholly in harmon with the view that the meridional circulation plays a major role in the total polewar transport of energy.

In addition to the circulation discussed above there is, on general admissio a meridional circulation entirely within the tropical zone. One gram of air risin from 850 to 220 mb at the equator and descending in latitude 25° would, fro the figures in Table VIII, release only 1·1 $\times$ 10^7 ergs. This is only one fifth of th energy release per gram in the interzonal circulation.

(*d*) *Seasonal variations*

It is apparent from Tables V and VI that the eddy fluxes of enthalpy and, t a lesser extent, of water vapour (latent heat) show a marked and regular annua variation. Fig. 1 shows the sum of the two and also the estimated values (Baur an Phillips 1935) of the total energy flux at certain fixed times of year ; this estimat comprises the amount required to balance the radiation amounts, corrected fo the known rate of storage of heat in the various zones at the different seasons. Th months are indicated as if all the data applied to the northern hemisphere. Th curves sketched to fit the two sets of values are broadly in phase. The most signi ficant feature of the comparison is that, while the eddy flux may approximatel fulfil the requirement in summer (bearing in mind the nature of both estimates) it is quite inadequate in winter. The natural inference would be that the mea meridional flow across these latitudes is slight in summer and marked in winte a view which has already been advanced from dynamical reasoning (Priestley 1950)

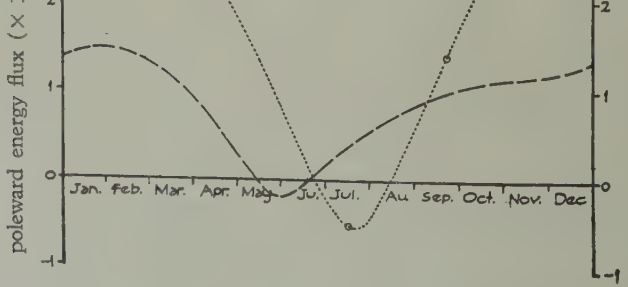

Figure 1. Poleward flux of energy across latitude 30-35°. The hatched curve shows the observe eddy flux, the dotted line the total flux according to Baur and Phillips. Months are indicated as all the data applied to the northern hemisphere.

It remains to examine whether considerations of momentum balance further onfirm this view. The results in Table VII give no indication of a marked annual ycle in the eddy flux of momentum. The annual variation in the total source rength of angular momentum between latitudes $\pm 30°$ to 35° may be assessed by ie method described in Section 1(c), and is represented by the following monthly otals :

Jan.	April	July	Oct.	Annual
0·19	0·19	0·15	0·16	$2{\cdot}0 \times 10^{34}$ g cm^2 sec^{-1}

his is quite a small relative variation. On the other hand it is found by the same ethod that the total sink strength N. of 30°N. experiences an approximately five-old increase from July to January. The corresponding figures for the southern ink have not been worked out, but the indications are that its percentage variation s less than the northern though greater than that of the source strength in the ropics.

The storage term remains to be considered. According to Munk and Miller 1950) the angular momentum of the whole atmosphere increases by 9×10^{32} cm^2 sec^{-1} from July to January, a rate of monthly storage or loss of $1{\cdot}5 \times 10^{32}$ g m^2 sec^{-1} or about one tenth of the total source strength at any time. It follows hat, after short-period fluctuations have been smoothed out, the total strengths of he two sinks combined must follow, to within about 10 per cent. throughout the ear, the total strength of the source. Recalling the absence of annual variation in he figures for eddy flux, the most simple and probable interpretation of this set of facts is that the mean toroidal flux of angular momentum across the separating atitudes is primarily a winter phenomenon.

This test in itself, however, does not decide whether the variation is accounted or entirely by the known winter strengthening of upper jetstream winds in the ounding zone, or whether a corresponding strengthening of $\overline{V}_1$ is an associated nd contributing factor. The evidence from the energy and other dynamical con-iderations supports the latter alternative. If this is so, it would imply a positive easonal correlation between $\overline{V}_1$ and $\overline{U}$, which would seem entirely plausible on dynamical grounds ; the contribution to the momentum flux would then be greater han that from a seasonally constant $\overline{V}_1$ with the same annual mean value.

(e) *Vertical distribution of the eddy flux*

Brief mention may be made of the variation of eddy flux with height. The haracteristic vertical distribution of eddy flux of enthalpy noted for Larkhill Priestley 1949) reappears in latitudes 30° - 35°. In the lowest layers the heat low is polewards, but this falls off with height and in many cases even becomes eversed in the middle or upper troposphere, while in the stratosphere the flow is gain uniformly polewards and of substantial amount. In contrast, for North America, White (1950) finds a roughly constant poleward flux up to and including 500 mb and a marked reversal in the lower stratosphere, features which he regards as in line with normal synoptic experience. Elsewhere, however, experience indi-cates the frequent occurrence of systems with marked thermal asymmetry in which

the air in the equatorward upper current ahead of the upper tropospheric ridge is notably warmer than that in the poleward current behind. One is tempted to offer a generalization that in regions governed by systems moving eastwards fairly rapidly the structure is such that the distributions found here and for Larkhill are typical but that this is not so where the systems are anchored or more slowly moving and show a greater measure of thermal symmetry. This would conform with the dynamic principle that the eastward ' wave velocity ' of the surface-pressure field is greater the greater the phase difference between this field and that of mean tropospheric temperature.

The eddy flow of momentum, and *ipso facto* of angular momentum, also shows a characteristic vertical distribution which appears in each station-year as a whole, and also for the mean of all stations in each season. At 900 mb $\overline{V'U'}$ is uniformly negative. By 500 mb it is uniformly positive but still small, a marked positive peak being reached at the level of 300 to 200 mb. In each case the contribution from the lower 500 mb of the atmosphere was such a small part of the total that, in problems concerned with total transfer, one might without serious error neglect it altogether. This result conveys the strongest possible warning as to the limitations of global dynamic research based on charts displaying conditions in the lower half of the atmosphere.

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A hemispherical study of the atmospheric angular-momentum balance

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SUMMARY

An attempt is made to determine the meridional eddy transport of angular momentum for the northern hemisphere in the vicinity of 30°N. latitude from upper-wind observations for a period of six months. Two somewhat different methods of evaluation give essentially comparable results and indicate the presence of eddy transports which are ample to compensate for frictional losses of angular momentum farther north. No indications of the presence of a mean meridional circulation of the strength and character needed to produce a sensible northward transport of angular momentum are found.

1. INTRODUCTION

In a previous communication the first author of the present paper (Starr 1951) reported the results of a preliminary study of the eddy transport of angular momentum across various latitude circles, evaluated from upper-wind data over the North American sector of the northern hemisphere for the month of February 1949. As was therein pointed out, the subject is of such basic importance for the systematic investigation of the processes of the general circulation that no effort should be spared to explore further the mechanisms which may be operative in the atmosphere to effect the needed transfer of angular momentum. Accordingly, a second provisional investigation of greater scope in certain respects was entered upon and is presented below.

The pilot study already mentioned is deficient in many important respects. Although the lack of proper observations is one of the most serious limiting factors in any endeavour to eliminate these deficiencies, it appears that by concentrating attention on the zonal belt in the vicinity of 30°N. latitude existing data permit rough extensions in the following directions : (a) Although the network of radio wind-stations becomes sparse, the inclusion of relatively few pilot-balloon stations makes it possible to work with a latitude zone completely encircling the earth. (b) The limitation to a single latitude zone makes possible the evaluation of data over a much longer period of time without a proportional increase in the labour involved. It was thus practical to deal with mean conditions over a period of six months. (c) The consideration of a complete latitude belt renders it possible to examine the data in regard to direct indications concerning the nature of mean meridional circulations. (d) With relatively large amounts of data it was found desirable to apply statistical tests of significance to the results. Such tests probably do not furnish infallible criteria of the soundness of conclusions, but nevertheless may be considered as supporting evidence in those cases where they are fulfilled. They may thus be looked upon more as giving necessary rather than sufficient conditions for acceptance of results.

The latitude zone in the vicinity of 30°N. is of special interest for studies of the angular-momentum balance of the atmosphere. Since this zone is roughly the dividing line between the surface westerlies to the north and surface easterlies to the south, the largest meridional transports of angular momentum in the northern hemisphere should be found here, on the average.

2. Observational material

The string of 16 key stations used for the study is listed in Table I. Since reports were often missing from the key stations, it was decided that alternative stations in the vicinity of certain of the key stations should be used in order to increase the amount of observational material. These alternative stations are entered in italics below the key stations in the table.

TABLE I. List of key stations (numbered) and alternate stations (italics) used in study

Station	Latitude* (N.)	Longitude	Altitude (ft)	Type
1. Qrendi	35° 50′	14° 27′E.	442	radio wind
2. Bahrein	26 16	50 38 E.	1	,,
3. Hyderabad	25 23	68 25 E.	95	pilot balloon
Fort Sandeman	31 21	69 27 E.	4613	,,
Peshawar	34 01	71 35 E.	1165	,,
Ambala	30 23	76 46 E.	892	,,
Bikaner	28 00	73 18 E.	735	,,
Johdpur	26 18	73 01 E.	735	,,
Ahmedabad	23 02	72 35 E.	164	,,
4. Dibrugarh	27 28	95 55 E.	384	,,
Tezpur	26 37	92 47 E.	259	,,
Cooch Bihar	26 20	89 27 E.	157	,,
Asansol	23 41	86 59 E.	413	,,
Gaya	24 45	84 57 E.	364	,,
Calcutta	22 32	88 20 E.	20	,,
Gorakhpur	26 45	83 22 E.	253	,,
5. Tokyo	35 33	139 46 E.	37	radio wind
Kagoshima	31 34	130 33 E.	18	,,
Itazuke	33 35	130 27 E.	20	,,
6. Midway	28 13	177 21 W.	0	,,
7. Honolulu	21 20	157 55 W.	15	,,
8. Weather Ship (a)	30 00	140 00 W.	0	,,
9. Santa Maria	34 56	120 25 W.	238	,,
10. Big Spring	32 14	101 30 W.	2537	,,
El Paso	31 48	106 24 W.	3916	,,
11. New Orleans	30 00	90 16 W.	30	,,
Lake Charles	30 13	93 09 W.	32	,,
12. Miami	25 49	80 17 W.	12	,,
Tampa	27 58	82 32 W.	11	,,
13. Kindley Field	32 22	64 40 W.	16	,,
14. Weather Ship (b)	35 00	40 00 W.	0	,,
Weather Ship (c)	34 00	52 00 W.	0	,,
15. Lagens	38 45	27 05 W.	171	,,
16. North Front	36 09	05 21 W.	26	,,

* Average latitude of key stations 31°

Wind reports for the levels 2, 6, 10, 14, 20, 25, 30, 35, 40, 45, and 50 thousand ft were taken from the data tabulations of the *Daily Series Synoptic Weather Maps* prepared by the U.S. Weather Bureau, in cooperation with the Army, Navy and Air Force for the period from 1 Feb. 1949 to 31 July 1949 for the hour 0300 GMT for each day. The frequency of available observations at the various levels is given in Table II.

TABLE II. PERCENTAGE OF TOTAL OBSERVATIONS WHICH ARE AVAILABLE BY STATION AT EACH LEVEL

	Elevation in thousands of ft										
Station	2	6	10	14	20	25	30	35	40	45	50
Qrendi	3·4	3·7	3·2	3·7	3·2	3·4	4·3	1·5	2·2	1·2	1.6
Bahrein	4·1	4·2	3·6	4·1	3·6	3·5	4·4	0·7	5·6	0·6	7·6
Hyderabad	8·0	0·0	7·7	0·0	7·3	4·8	2·2	0·2	0·0	0·0	0.0
Dibrugarh	8·0	0·0	6·2	0·0	3·4	2·2	1·2	0·0	0·0	0·0	0·0
Tokyo	8·2	8·7	7·8	9·2	8·7	9·9	10·9	13·1	11·4	13·0	10·8
Midway	6·2	5·9	4·5	5·1	4·3	3·8	3·8	4·7	3·9	4·8	4·7
Honolulu	7·4	8·0	7·0	8·6	7·8	8·1	8·7	11·0	10·5	12·6	13·2
Weather Ship (a)	7·1	7·3	6·3	7·4	6·5	6·7	6·7	7·3	6·2	7·1	6·3
Santa Maria	8·2	8·9	7·8	9·5	8.7	9·6	9·9	12·4	13·3	15·2	16·3
Big Spring	0·0	8·9	7·7	9·4	8·2	8·8	8·5	9·9	9·9	11.2	10.6
New Orleans	8·0	8·9	7·6	9·0	8·2	8·1	7·8	9·4	8·4	7·8	5·8
Miami	8·3	9·0	7·9	9·4	8·6	9·5	9·7	12·3	11·9	13·1	11.0
Kindley Field	6·1	6·4	5·6	4·5	3·8	2·8	2·3	1·7	1·0	0·9	0.3
Weather Ship (b)	5·8	7·0	6·2	7·4	6·6	6·8	7·2	8·9	8·7	7·8	7.6
Lagens	6·1	6·6	5·6	6·5	5·7	5·7	5·6	5·9	4·0	3·6	1.0
North Front	5·0	6·3	5·3	6·4	5·3	6·1	7·1	1·1	3·1	1·2	3.4

Surface winds were excluded from the study, since they can be too easily rendered unrepresentative by local orographic influences. No attempt was made to correct or exclude any original data where doubtful even though in some cases such procedures might have improved the results. This course was followed in order to maintain objectivity in the computations. Each wind observation was resolved into the eastward component u and northward component v, tabulated in m sec^{-1}. From these quantities the transports of angular momentum were computed by methods discussed presently.

3. General considerations

The total meridional transport of absolute angular momentum across a latitude circle may be thought of as the sum of the transport of relative angular momentum and the transport of angular momentum due to the earth's rotation. Since there is no net meridional transport of mass in the atmosphere in the long run, the transports of angular momentum must be brought about by exchange processes. Furthermore, since the angular momentum due to the earth's rotation per unit mass is essentially constant with respect to elevation in the atmosphere at a given latitude, the net effect of all exchange processes must be negligible as far as this component is concerned. It therefore follows that we may omit this contribution and deal only with the transport of relative angular momentum.

Also, owing to the fact that the distance from the earth's axis varies percentually only very slightly with elevation in the atmosphere at the given latitude, the transport of relative angular momentum is given by the transport of relative linear momentum except for a constant factor. For these reasons the transport of relative linear momentum will be considered first.

The total transport of relative linear momentum τ across a given complete latitude circle at a given level over a time interval t_1 may be expressed as

$$\tau = \int_0^{t_1} \oint \rho u v dx dt \simeq \rho^* \int_0^{t_1} \oint u v dx dt = \rho^* \overline{[uv]} L t_1 \quad . \quad . \quad (1)$$

where ρ is the density, ρ^* is an appropriate average density, dx is an element of linear distance and the square brackets denote a space average over the length L of a complete latitude circle and the bar denotes a time average over the interval t_1. In (1) the assumption is made that at a constant level the effects of density variations may be neglected. The feasibility of making this assumption is considered in a subsequent section. The quantities u, v may be separated into the mean motion $[u]$, $[v]$ and the deviations from the means u', v' so that

$$u = [u] + u' \quad ; \quad v = [v] + v' \quad . \quad . \quad . \quad (2)$$

in which the square brackets again denote instantaneous space averages. It follows that

$$[uv] = [u][v] + [u'v'] \quad . \quad . \quad . \quad . \quad (3)$$

If Eq. (3) is integrated over the given time interval t_1, and averaged, one obtains

$$\overline{[uv]} = \overline{[u]}\,\overline{[v]} + \overline{[u'v']} \quad . \quad . \quad . \quad . \quad (4)$$

where the bars again denote time averages. In a manner analogous to that used in writing (2) it is possible to write expressions for $[u]$, $[v]$ of the form

$$[u] = \overline{[u]} + [u]' \quad ; \quad [v] = \overline{[v]} + [v]' \quad . \quad . \quad (5)$$

where $[u]'$, $[v]'$ denote deviations of the space means from the space-time averages. It follows from (1), (4) and (5) that

$$\frac{\tau}{\rho^* L t_1} \simeq \overline{[uv]} = \overline{[u]}\,\overline{[v]} + \overline{[u]'[v]'} + \overline{[u'v']} \quad . \quad . \quad (6)$$

since the quantities $\overline{[u]}'$, $\overline{[v]}'$ vanish.

Within the limits set by the assumption concerning the horizontal uniformity and constancy of the density, Eq. (6) may be given a simple interpretation. Since $\overline{[v]}$ is a measure of the net meridional flow of air during the given time interval, the first term on the right measures the contribution of the *mean* meridional circulation to the momentum transport. This will be called the *transport of the first species*. Such net air flow over any extended period generally implies a return flow at some other level although the effects of net meridional mass shifts are also included in the transport of this type.

Since $[v]$ is the instantaneous meridional circulation (which may or may not be compensated for at other levels), it is possible that the temporal fluctuations of $[v]$

re correlated with $[u]$ and thus produce a contribution to the transport. This may appen even if $\overline{[v]}$ is zero, *i.e.*, if the instantaneous meridional circulation is merely n oscillation producing no net transport of air during the interval t_1. This action is epresented by the second term on the right, and may be considered as a certain ype of eddy transport. This will be called the *transport of the second species*. For nore or less obvious reasons it would be of considerable interest to know whether his process is of importance in the atmosphere.

Finally, since $[u'v']$ is dependent on the instantaneous spatial correlation of and v along the length of the latitude circle, the last term gives the eddy transport ue to horizontal-exchange processes during the interval t_1. This may be named he *transport of the third species*.

In order to ascertain the relative importance of the three terms, one may note hat the first, $\overline{[u]}\,\overline{[v]}$, is, at least in principle, obtainable directly from observed alues of u and v. The second $\overline{[u]'\,[v]'}$ is obtainable from (4) and (6) being equal o $\overline{[u]\,[v]} - \overline{[u]}\,\overline{[v]}$. The third is given by (4) directly as $\overline{[uv]} - \overline{[u]}\,\overline{[v]}$. Thus he basic quantities to be computed are $\overline{[u]}$, $\overline{[v]}$, $\overline{[u]\,[v]}$ and $\overline{[uv]}$.

4. Computation of transports

As an approximation the instantaneous space averages $[u]$, $[v]$, $[uv]$ were taken o be the arithmetic means of the values for the several stations reporting at a given evel on a given day. Days on which only one station reported an observation were excluded. The maximum number of observations on a given date at a given evel was sixteen, this being the total number of observation points.

The time means represented by the basic quantities $\overline{[u]}$, $\overline{[v]}$, $\overline{[u]\,[v]}$ and $\overline{[uv]}$ were then formed by taking the arithmetic means of the daily quantities for the number n of days for which data were present. The results are tabulated in columns 1 o 9 in Table III.

In order to secure some measure of statistical reliability of the time averages, confidence limits were calculated and are included for various quantities in the body of the table. These limits are defined as twice the standard error and indicate approximately the 95 per cent. confidence level. The formula used is

$$\sigma_{\bar{y}} = \frac{\sigma(y)}{\sqrt{n}} \qquad . \qquad . \qquad . \qquad . \qquad (7)$$

where $\sigma_{\bar{y}}$ is the standard error of the mean of any quantity y, $\sigma(y)$ is the sample standard deviation of y and n is the number of cases in the sample (number of days).

The space-time means $\overline{[u]}$, $\overline{[v]}$, $\overline{[uv]}$ represent averages over the number of days involved in each case of the daily space averages, $[u]$, $[v]$, $[uv]$ respectively, as has already been explained. As such these space-time averages are formed by an equal weighting of the daily values. Owing to the fact that the daily values $[u]$, $[v]$, $[uv]$ are based upon a number of observations which may vary from 2 to 16, the equal weighting introduces a certain amount of artificiality in the procedure. It is therefore of interest to compare the results already mentioned with the results

TABLE III. NUMERICAL ANALYSIS OF DATA. THE LEVELS ARE GIVEN IN THOUSANDS OF FT. ALL VELOCITIES ARE IN M SEC^{-1}. INTERNAL CONSISTENCY OF THE FIGURES GIVEN IS LIMITED BY ROUNDING-OFF APPROXIMATIONS.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[uv]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u]'}\,\overline{[v]'}$	$\overline{[u'v']}$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
50	+ 14·0	− 0·07 ± 0·76	+ 3 ± 19	+ 30 ± 34	− 1	+ 4	+ 27 ± 21	149	+ 12·3	− 0·26	+ 21	+ 24	+ 0·14	604
45	+ 18·0	− 0·45 ± 1·02	− 6 ± 27	+ 43 ± 46	− 9	+ 3	+ 48 ± 24	162	+ 16·6	− 0·10	+ 49	+ 51	+ 0·23	764
40	+ 18·8	− 0·25 ± 0·93	− 4 ± 24	+ 30 ± 31	− 5	+ 1	+ 34 ± 16	178	+ 17·0	+ 0·14	+ 32	+ 30	+ 0·12	1032
35	+ 17·2	− 0·19 ± 0.75	0 ± 19	+ 40 ± 29	− 3	+ 4	+ 39 ± 19	180	+ 16·0	− 0·12	+ 38	+ 40	+ 0·17	1177
30	+ 14·2	+ 0·11 ± 0·57	− 1 ± 13	+ 33 ± 15	+ 2	− 3	+ 35 ± 11	181	+ 13·2	+ 0·02	+ 34	+ 33	+ 0·18	1628
25	+ 11·1	− 0·12 ± 0·86	− 3 ± 7	+ 17 ± 10	− 1	− 2	+ 19 ± 7	181	+ 10·9	− 0·03	+ 17	+ 18	+ 0·14	1798
20	+ 8.8	+ 0·20 ± 0·34	+ 1 ± 4	+ 17 ± 7	+ 2	− 1	+ 17 ± 5	181	+ 8·3	+ 0·11	+ 15	+ 15	+ 0·15	2038
14	+ 6·1	+ 0·29 ± 0·29	+ 3 ± 2	+ 12 ± 4	+ 2	+ 1	+ 9 ± 4	181	+ 6·0	+ 0·19	+ 10	+ 9	+ 0·14	1888
10	+ 3·4	+ 0·19 ± 0·20	+ 1 ± 1	+ 6 ± 2	+ 1	0	+ 6 ± 2	181	+ 3·3	+ 0·19	+ 6	+ 5	+ 0·09	2286
6	+ 1·3	+ 0·86 ± 0·35	+ 1 ± 1	+ 4 ± 2	+ 1	0	+ 3 ± 2	181	+ 1·3	+ 0·68	+ 4	+ 3	+ 0·08	2008
2	− 0·3	+ 0·03 ± 0·22	0 ± 0	+ 3 ± 2	0	0	+ 3 ± 1	181	− 0·3	+ 0·03	+ 4	+ 4	+ 0·11	2172
Integral (10^7 CGS units)				+ 14·2	− 0·1	+ 0·2	+ 14·1				+ 13·9	+ 13·7		Sum 17,395

of another method of estimating the space-time averages which is free from the particular objection mentioned but which may contain artificialities of a different nature. The alternative procedure used consists of summing all the individual observations at a given level for the entire six-month period and dividing this sum by the total number of observations N (column 15). The space-time averages so computed are indicated in Table III by curly brackets, *e.g.*, $\{u\}$. Thus the quantities in columns 10, 11, 12, are to be compared with columns 2, 3, 4 respectively.*

According to Eq. (6) the quantity $\overline{[uv]} - \overline{[u]}\,\overline{[v]}$ gives the sum of the transports of the second and third species, *i.e.*, the sum of the quantities in columns 7 and 8 in the table. The analogous transport $\{uv\} - \{u\}\{v\}$ evaluated by the alternative method is given in column 13. It is to be noted that the two methods of evaluating the space-time averages would become identical in those cases where the number of individual observations is the same from day to day (a circumstance not present in the data used).

According to the definitions of the quantities involved, it follows that

$$\{uv\} - \{u\}\{v\} = r(u,v) \cdot \sigma(u) \cdot \sigma(v) \quad . \quad . \quad . \quad (8)$$

where $r(u, v)$ is the coefficient of linear correlation between u and v, and $\sigma(u)$, $\sigma(v)$ denote the standard deviations of these wind components. Eq. (8) may be used to determine r for the N observations. For the sake of general interest the several values of r are listed in column 14 of Table III.

Vertical integrals of various quantities weighted by the density are given at the foot of the table. These were computed as follows : giving the hydrostatic relation we may write that

$$\int_{z_1}^{z_2} \rho(\ )dz = -\frac{1}{g}\int_{p_1}^{p_2}(\ )dp \quad . \quad . \quad . \quad . \quad (9)$$

where z is height, g is the (constant) acceleration of gravity, p is pressure. The right-hand side of (9) was evaluated by numerical integration using the standard-atmosphere pressures corresponding to the several levels used. The effect of this use of the standard atmosphere is to make ρ^* in Eq. (1) equal to the standard-atmosphere density. Following a method due to Priestley (1949) the integrals obtained give the transports per unit length of the latitude circle from 2,000 ft to 50,000 ft.

5. Discussion of numerical results

The results obtained contain a number of points which are of first concern in studies of the general circulation. Among these are the following :

(a) Upon comparison of the transports of the third species given in column 8 of Table III with columns 6 and 7, the figures obtained show about one order of magnitude difference. Moreover, the figures for the third species are all positive with confidence limits which in every instance exclude zero by a substantial margin.

* This alternative method is the one used previously in the North American pilot study. In view of the fact that in the pilot study only a segment of a latitude circle was sampled it was not feasible to make a distinction between the transport of the second and third species.

The figures for the other two species on the other hand suggest that their contribution is sufficiently small so that the analysis cannot give reliable estimates (or perhaps even give the correct sign). It is thus strongly suggested that horizontal-exchange processes are the dominant mechanism for the poleward transport of momentum from the easterlies to the westerlies.

(b) The sums of the items in column 7 with those in column 8 are roughly comparable to column 13 showing that somewhat different computing methods give essentially the same results. The same applies to columns 5 and 12 except perhaps at 50,000 ft. Since the transports of the third species appear to be dominant, the values in columns 5, 8, 12, 13, should all be comparable, which is roughly the case.

(c) The transports of the third species increase with altitude, showing a probably spurious irregularity at 40,000 ft, then decrease at 50,000 ft. The general shape of the profile is similar to that obtained by Starr (1951) from North American data for 35°N. latitude during winter. It is possible that a seasonal vertical shift in the level of maximum transport exists. In that case the present data approximate a mean from winter to summer. Individual monthly values are not included, since their reliability was not deemed sufficient to warrant their detailed study. It may however be noted that of the 66 of such monthly values calculated 64 were positive in sign, one was zero and one was negative (May, at 50,000 ft).

(d) Although many questions arise, as discussed in a subsequent section, the figures for $[\overline{v}]$ in column 3 would seem to show that such mean meridional circulations as may be present are so small that the analysis lacks sufficient statistical resolving power to detect them. The values themselves are remarkably small, being in most cases not statistically different from zero. The possible exceptions are the small positive values at low levels downward from 20,000 ft where the necessary condition set by the confidence limits borders on fulfilment but is surpassed only at 6,000 ft. For reference later the values of $[\overline{v}]$ calculated for the colder months, Feb., Mar. and Apr. taken together, become larger and fulfil the confidence limits (*i.e.*, zero is excluded) for the 6,000, 10,000, and 14,000 ft levels. Furthermore, it is worthy of note that although the analogous quantity $\{v\}$ given in column 11 generally has still smaller (numerical) values except at 50,000 ft, at lower levels the values are again consistently positive, being in closer agreement with $[\overline{v}]$ than elsewhere.

(e) The vertical integrals at the foot of the table reflect what has been said concerning the values for the individual levels. Thus the integral for the transport of the third species in column 8 far overshadows the integrals for the first and second species in columns 6 and 7, making it about equal to the integral for the total in column 5. The alternative method gives integrals (columns 12 and 13) which are comparable to those for the first method. If the value of the integral for column 8 is multiplied by the length of the latitude circle and by the distance from the earth's axis at 31°N. latitude so as to obtain the transport of angular momentum, one obtains 26×10^{25} CGS units for this quantity. An estimate for the polar cap north of 31°N. of the *normal* average drain of angular momentum due to surface effects for the six-month period made by Widger (1949) and White (1949) is 15×10^{25} CGS units. As noted by Starr (1951) these determinations of surface torques probably represent a substantial underestimate.

6. Critical remarks

The computations outlined above involve many questions as to the validity of the procedures used. The more important of these are the following.

(a) The location of the observation points might be biased so as to sample unduly conditions to the east or to the west of troughs (or ridges) in the streamline patterns. If this effect were marked, the expectation would be that the net meridional velocities $\overline{[v]}$ and $\{v\}$ should be large. These quantities on the whole turn out to be surprisingly small, suggesting a rather random sampling with respect to the troughs and ridges. The extension of the averaging over changing seasonal patterns doubtless aids in the achievement of such desired randomness.

(b) The geographic sampling with respect to longitude leaves much to be desired especially over southern Asia and the Pacific. Little can be done about this deficiency since only a few stations are available. However, the effects of sparse data become most marked only at the higher levels as shown in Table II.

(c) The variations in latitude of the stations introduce an undesirable element. This is especially true of Honolulu and Lagens. The seriousness of this factor cannot be immediately appraised except to say that the results should perhaps be looked upon as an average over a small belt of latitude as well as over longitude and time.

(d) The radio wind soundings are more likely to reach high elevations under relatively light-wind conditions. This effect is perhaps more marked than is commonly realized, as pointed out by Lorenz (1950). Evidence of the presence of this factor is found in that the several values of $\overline{[u]}$ are consistently higher than the corresponding values of $\{u\}$ except at 2,000 and 6,000 ft. This is to be explained by noting that evidently those days on which the upper westerlies were strong gave relatively few soundings penetrating to high levels. In calculating $\overline{[u]}$ such days were given the same weighting as other days. On the other hand, in calculating $\{u\}$ the individual soundings were weighted equally, so that the effects of the few soundings showing abnormally high winds were minimized. The presence of of this type of selectivity probably brings about an underestimate of the momentum transports. Probably the slightly smaller transports secured by the second method are likewise a reflection of this factor.

The use of the two pilot-balloon observing points in southern Asia probably increases the selectivity effects, since these soundings are even more subject to such shortcomings.

(e) The values of $\overline{[v]}$ and $\{v\}$ may not actually represent a mass flux, *i.e.*, $\overline{[\rho v]}$ and $\{\rho v\}$ might be substantially zero even though the former quantities have non-zero values. This can happen only if ρ and v are properly correlated. Such correlations seem to exist actually, although from order-of-magnitude considerations it would be difficult to interpret the high values of $\overline{[v]}$ and $\{v\}$ at the 6,000 ft level as representing a volume but not a mass transport. On the other hand, these particular values may be over-estimates since reports from the two pilot-balloon stations in southern Asia were not available for this level, as indicated in Table II. That a contribution to $\overline{[v]}$ and $\{v\}$ might be present from a density correlation is rendered more plausible by the fact that these velocities were more definitely positive and larger in magnitude below 20,000 ft during the colder months Feb.,

Mar. and Apr., as mentioned earlier. It is during these months that differential advection effects might be expected to be more intense.

(f) As indicated earlier a constant standard-atmosphere density was used as an approximation in Eq. (1). In order to secure a measure of the error thus introduced, a limited study was made over the North American sector in the vicinity of 35° N. latitude at the 20,000 ft level for Feb. 1949. Over this region thermodynamic soundings were available for each wind observation used so that the eddy transport could be evaluated both with and without the approximation mentioned, on otherwise identical data. It was found that the difference amounted to about one per cent. Further checks at other levels should be made concerning this point.

(g) It may seem paradoxical that although the confidence limits are poorly satisfied by the items $\overline{[uv]}$ and $\overline{[u]}\,\overline{[v]}$ in Table III, they are very well satisfied by the differences of these quantities $\overline{[u'v']}$ in column 8. This can happen only if the daily values of $[uv]$ and $[u]\,[v]$ are correlated, which evidently is the case, thus perhaps implying a compensation of errors when the difference is taken.

(h) The contributions to the transport due to the residual mass of the atmosphere above 50,000 and below 2,000 ft have not been evaluated. Doubtless the contribution from the layer below 2,000 ft is small on the basis of the fact that the wind velocities become smaller here. The situation above 50,000 ft needs more study. With existing data some indications may be obtained up to, let us say, 60,000 ft although a much longer period of time is necessary to secure statistical reliability. It would nevertheless appear from casual examination of higher-level data that the eddy transports continue to decrease above the jet-stream level.

(i) The six-month period examined represents conditions during a period of warming of the northern hemisphere, *i.e.*, conditions from winter to summer. Some reservation must be made concerning various details as to their representativeness for longer periods. It is hoped that a study similar to the present one will be available at a later date covering a comparable period from summer to winter.

7. Meteorological implications

Although the results obtained in this study need to be checked by further extensive research involving other and longer periods of time as well as a better network of reporting stations, it is nevertheless very interesting to discuss the import of the tentative indications obtained upon scientific thought concerning the general circulation, assuming that there exists a rough correspondence between the true facts and the picture of conditions as here derived from observations. On this understanding the following statements may be made.

In the first place the existence of the strong transports of the third species reveals a very characteristic and deep-seated feature of the general circulation. This bears out the contention of Jeffreys (1926) communicated in the pages of this Journal some twenty-five years earlier and more recently re-examined by Starr (1948), Bjerknes (1948), Van Mieghem (1949) and Mintz (1949). The contention has already received considerable corroboration through the studies of Widger (1949) and also through the pilot study made by Starr (1951).

Since these transports of the third species appear to be ample in magnitude to compensate for frictional and other surface losses farther to the north, it is difficult

to escape the conclusion that *ipso facto* the other modes of angular momentum transfer are of relatively minor importance. This is indicated, as it should be, by the relative insignificance of the transports involving mean and instantaneous hemispherical meridional circulations, *i.e.*, the transports of the first and second species. This circumstance together with the lack of evidence for the presence of a *direct* mean meridional circulation of the order of 1 m sec^{-1} (which is required) would render untenable certain suggestions of Palmén (1950) concerning a driving mechanism for the general circulation based on large-scale vertical convection between the more tropical and more polar regions.

On the other hand these circumstances would suggest that theories for the mechanism of the general circulation should follow along lines such as those proposed by Rossby (1947) or by Kuo (1950) in so far as these theories specify horizontal or quasi-horizontal exchange processes as being the dominant factor in the maintenance of the zonal westerlies. These contentions have received other support from the experimental work of Fultz (1948).

As already mentioned the slight northward values of $\overline{[v]}$ and $\{v\}$ may represent a volume rather than a mass transport. If this is true it would support the proposal made by Starr (1949, 1951) that the kinetic-energy balance of the atmosphere is capable of fulfilment in terms of horizontal-exchange processes without mean meridional circulations involving mass flux across latitude circles at individual levels.

As a general comment upon the type of meteorological research exemplified by the present study, it is appropriate to point out that *sufficient* demonstrations of the validity of conclusions based on the examination of data are in general difficult to attain. It is rather through the amassing of supporting evidence from many studies carried through by various investigators using independent data and differing methods (where permissible) that a certain measure of confidence is ultimately established. The writers therefore hope that this discussion will encourage further work of this general nature.

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The quasi-geostrophic advective wave in a baroclinic zonal current

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SUMMARY

An idealised advective wave is studied on the assumption of geostrophic advection of temperature. There is a genuine instability of zonal motion due to this factor and the wave form is similar to thermally asymmetrical depressions and anticyclones. The speed of the unstable waves is $\bar{U} - U_c/2$ rather than the $\bar{U} - U_c$ of barotropic theory ($\bar{U}$ the mean zonal wind, U_c the Rossby westward component due to variation of Coriolis parameter).

Waves become stable when (approximately) the root-mean-square zonal thermal wind becomes equal to U_c. This is near the stationary wave length. There are two types of stable waves, the thermally in-phase and the thermally antiphase waves. The antiphase waves move eastward relative to the surface zonal flow, the in-phase waves are retrograde.

The wave characteristics depend on the thermal-wind conditions, the surface zonal wind is simply additive to the wave speed.

The results support the author's development theory. They do not encourage much optimism in the direct application of barotropic concepts to the baroclinic atmosphere.

1. INTRODUCTION

We shall discuss the characteristics of a wave disturbance to zonal flow which satisfies the conditions that :

(1) the vorticity equation is satisfied in the simple form appropriate for horizontal isobaric motion,

(2) the thermal field is modified by advection only,

(3) the advection and the vorticity are given by their geostrophic values.

The model is artificial but is of interest in connection with the author's " theory of development " (Sutcliffe 1947) and with the general technique of forecasting using thermal patterns (Sutcliffe & Forsdyke 1950).

Jaw (1946) dealt with a similar problem and obtained a criterion of instability having much in common with that derived below. The differences between the two treatments are however very considerable and it is not at once apparent why the results are in accord. It is perhaps necessary to state quite clearly that the results are claimed to be suggestive rather than established, as applying to the real atmosphere. We have Queney's (1950) recent warning that " approximate laws must not be used as starting relations in a sequence of operations where differentiations are involved " and Queney gave point to this dictum by showing that the neglect of vertical motion introduced a spurious instability into his perturbation wave-equation. Nevertheless it is believed that the type of instability which our analysis describes is synoptically real and that, in meteorology, the less rigorous analysis has often suggested ideas of proven practical value and paved the way for the more rigorous solution.

It will be of interest to compare the result with the well-known formula (Rossby

939) for the speed of a wave in a barotropic atmosphere moving zonally, namely :

$$\overline{U} - c - U_c = \frac{l^2}{\mu^2} \cdot \frac{c}{gH - (\overline{U} - c)^2} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\overline{U}$ = the zonal current velocity
c = the wave velocity
$U_c = \beta L^2/(4\pi^2)$ is the east-to-west velocity due to the variation with latitude of the Coriolis parameter
l = Coriolis parameter
$\beta = \partial l/\partial y$; y northwards
$\mu = 2\pi/L$
L = wave length
H = height of the homogeneous atmosphere

The formula is commonly quoted without the last term which is small except for really long waves and has been applied to the flow at 500 mb, at a " level of non-divergence," or to the mean zonal flow of the westerlies by various workers with, in some cases, empirically determined multiplying factors giving formulae of the type

$$\overline{U} - ac - bU_c = 0$$

where $\overline{U}$ is now the selected zonal velocity. More recently Charney (1949) has put forward the idea of the " equivalent barotropic atmosphere " as a basis for practical computation but this idea, in spite of some refinements, is not notably different from that of applying the barotropic formula to a selected level.

2. The geostrophic advective model

We consider a uniform zonal current defined by

U_0 = surface zonal wind = constant
U' = thermal zonal wind, a function only of height
v_0 = a variable meridional surface wind
v' = the variable meridional component of the thermal wind
$U = U_0 + U'$
$v = v_0 + v'$

As has been shown elsewhere, the assumption that the field of temperature is changed only by horizontal geostrophic advection is equivalent to assuming that the isotherms (or thickness lines) are advected with the surface geostrophic wind.

If ϕ is the geopotential thickness of the layer from pressure p_0 to pressure p

$$\frac{\partial \phi}{\partial t} + U_0 \frac{\partial \phi}{\partial x} + v_0 \frac{\partial \phi}{\partial y} = 0$$

$$\frac{\partial \phi}{\partial t} + lU_0 v' - lU' v_0 = 0$$

and differentiating with respect to x,

$$\frac{\partial v'}{\partial t} + U_0 \frac{\partial v'}{\partial x} - U' \frac{\partial v_0}{\partial x} = 0$$

Now give v_0 the functional form

$$v_0 = V_0 e^{i\mu(x-ct)} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

representing a sinusoidal wave disturbance of wave velocity c which may be real (stable waves) or complex (growing or decaying waves)

and $$v' = V' e^{i\nu(x-kt)}$$

The quantity V_0 is constant but may be complex. V' is a function of height only; it may be a complex quantity but if so is of constant argument.

The model is, then, of a wave disturbance of unlimited latitudinal extent. This is the simplest model as employed by Rossby in the early paper cited. Modifications to take account of latitudinal limits have been introduced into wave theory by several writers but the object here is to retain the maximum mathematical simplicity in order not to mask the simple physical conceptions. Many refinements can be introduced to obtain greater closeness to synoptic reality if this elementary analysis is thought to show sufficient promise.

Then $$-\nu k v' + \nu U_0 v' - \mu U' v_0 = 0$$

$$v' = \frac{\mu U' v_0}{\nu(U_0 - k)}$$

This is only possible if $\nu = \mu$, $k = c$

and $$\frac{v_0}{U_0 - c} = \frac{v'}{U'} = \frac{v}{U - c} \quad . \quad . \quad . \quad . \quad (3)$$

Thus the type of wave disturbance consistent with the assumptions is a three-dimensional wave with velocity and wave length independent of height. Thus if c is real v_0, v' and v are in phase or antiphase and the waves are similar to those discussed by Rossby (1942). If the surface pressure patterns and the thermal pattern are in phase (cold low and warm high) $U_0 - c$ and U' have the same sign and taking U' as positive

$$c < U_0$$

If the waves are in antiphase (warm low, cold high)

$$c > U_0$$

These results are synoptically acceptable.

Also if the amplitude of the surface isobars is B_0, of the isotherms is B' and of the upper contours is B,

then $$\frac{|v_0|}{B_0 U_0} = \frac{|v'|}{B' U'} = \frac{|v|}{BU}$$

$$\frac{|U_0 - c|}{U'} = \frac{B_0 U_0}{B' U'}$$

$$|U_0 - c| = B_0 U_0 / B'$$

us when the thermal wave has relatively large amplitude the difference ween U_0 and c is small, with relatively small amplitude it is large.

When c is complex we write

$$c = c_r + ic_i$$

$$c' \equiv c - U_0 = c_r' + ic_i$$

$$v_0 = -\frac{v'(c_r' + ic_i)}{U'} \quad . \quad . \quad . \quad (4)$$

When c_i is negative the real exponent in (2) is positive and the wave form is wing. From (4), the phase of the v' wave is then between 180° and 90° behind v_0 wave if c_r' is positive (wave velocity $> U_0$), and between 90° and 0° behind v_0 wave if c_r' is negative (wave velocity $< U_0$). Thus the growing wave has cold trough behind the pressure trough and, similarly, the decaying wave has cold trough ahead of the pressure trough.

These implications of the assumption of wave motion in accordance with rmal geostrophic advection are broadly in accord with the properties of many pressions and anticyclones in their early stages and suggest that the model has me realism.

Proceeding now to the dynamics we put ζ for the vorticity $\partial v/\partial x$ and from (3)

$$\zeta = i\mu v = i\mu v_0(U - c)/(U_0 - c)$$

The divergence*, from the vorticity equation, (valid in early stages or in long ves when vorticity is small compared with l) is

$$\begin{aligned} -l\mathrm{div}_p V &= d(\zeta + l)/dt \\ &= \partial\zeta/\partial t + U\partial\zeta/\partial x + v\partial l/\partial y \\ &= \mu^2 cv - U\mu^2 v + v\beta \\ &= v\mu^2(c - U + U_c) \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

If the flow were non-divergent at any level we should have the simple baro- pic wave equation

$$c - U + U_c = 0$$

However the condition in the atmosphere is that the vertical integral of diver- nce shall be equal to the hydrostatic pressure change below, thus :

$$-\frac{dp_0}{dt} = \int_0^{p_0} \mathrm{div}_p V dp$$

d integrating (5)

$$l\frac{dp_0}{dt} = \mu^2 \int_0^{p_0} v(c - U + U_c)dp$$

, with the bar representing the pressure-mean value in the vertical,

$$\frac{l}{p_0}\frac{dp_0}{dt} = \mu^2 \overline{v(c - U + U_c)}$$

* Isobaric co-ordinates, taking pressure as the independent variable defining position in the vertical, are used as in uthor's 1947 paper. The divergence is here the isobaric divergence of horizontal velocity; the two dimensional vorticity nilarly defined. The use of these co-ordinates is discussed also by Eliassen (1949).

or substituting for v in terms of v_0

$$\frac{l}{p_0}\frac{dp_0}{dt} = \frac{\mu^2 v_0 \overline{(c-U+U_c)(c-U)}}{c-U_0}$$

It may be shown, by analysis similar to that given by Rossby (1939), Holmboe (1945), or Charney (1947) that the left-hand member reduces effectively to the small right-hand term in Eq. (1) and for our present purpose we shall omit it from the discussion*. Then the condition is that

$$\overline{(c-U+U_c)(c-U)} = 0$$

that is
$$c^2 - c(2\bar{U} - U_c) + \overline{U^2} - \bar{U}\,U_c = 0$$

giving
$$c = \bar{U} - \frac{U_c \pm (U_c^2 - 4\bar{U}^2 + 4\overline{U^2})^{1/2}}{2}$$

Putting $c - U_0 = c'$, the excess of velocity over the surface zonal velocity, we may use the equivalent forms

$$c'^2 - c'(2\bar{U'} - U_c) + \overline{U'^2} - \bar{U'}\,U_c = 0$$

$$c' = \bar{U'} - \frac{U_c \pm (U_c^2 - 4\bar{U'}^2 + 4\overline{U'^2})^{1/2}}{2} \quad . \quad . \quad (6)$$

With a given thermal field and given wave length (or U_c) the wave velocity c', real or complex, is defined and the surface zonal velocity is simply additive. We shall prefer therefore to use the second form which keeps the baroclinity in prominence.

It is at once noted that the expression is quite different from the barotropic form

$$c' = \bar{U'} - U_c$$

and there is evidently no uniform level at which U' could be taken to make the barotropic form valid for varying thermal fields and varying wave lengths.

The barotropic form would of course emerge if $4(\overline{U'^2} - \bar{U'}^2)$ were negligible compared with U_c^2 but it is readily seen that this is practically never the case in situations of synoptic interest. If, for example, U' is taken to vary linearly with pressure fall in the vertical (or to increase to a maximum and decrease to zero linearly)

$$\overline{U'^2} = 4\bar{U'}^2/3$$

and
$$4(\overline{U'^2} - \bar{U'}^2) = \overline{U'^2} = 4\bar{U'}^2/3$$

Then
$$c' = \bar{U'} - \frac{U_c \pm (U_c^2 - \overline{U'^2})^{1/2}}{2} \quad . \quad . \quad . \quad 6(a)$$

and the barotropic formula only emerges when $\bar{U'}$ is small compared with c, *i.e.* when the thermal wind is itself negligible and the flow is truly barotropic. Such situations are hard to find in nature.

* The neglect of this term in this equation is equivalent to neglecting the vertical integral of divergence compared with the contributions at different levels above. It does not of course mean that surface pressure changes are not associated with the changing wave pattern.

3. Discussion of the wave-speed equation

It is clear from Eq. (6), or the approximate form 6(a), that there is a critical wave length for which the radical vanishes. For longer waves (larger U_c) and the same mean zonal thermal wind, the wave speed is real (with two values) and the wave is stable. For shorter waves (smaller U_c) the wave speed is complex; the real part (giving the real wave speed) is then

$$c_r' = \overline{U'} - U_c/2 \qquad c_r = \overline{U} - U_c/2$$

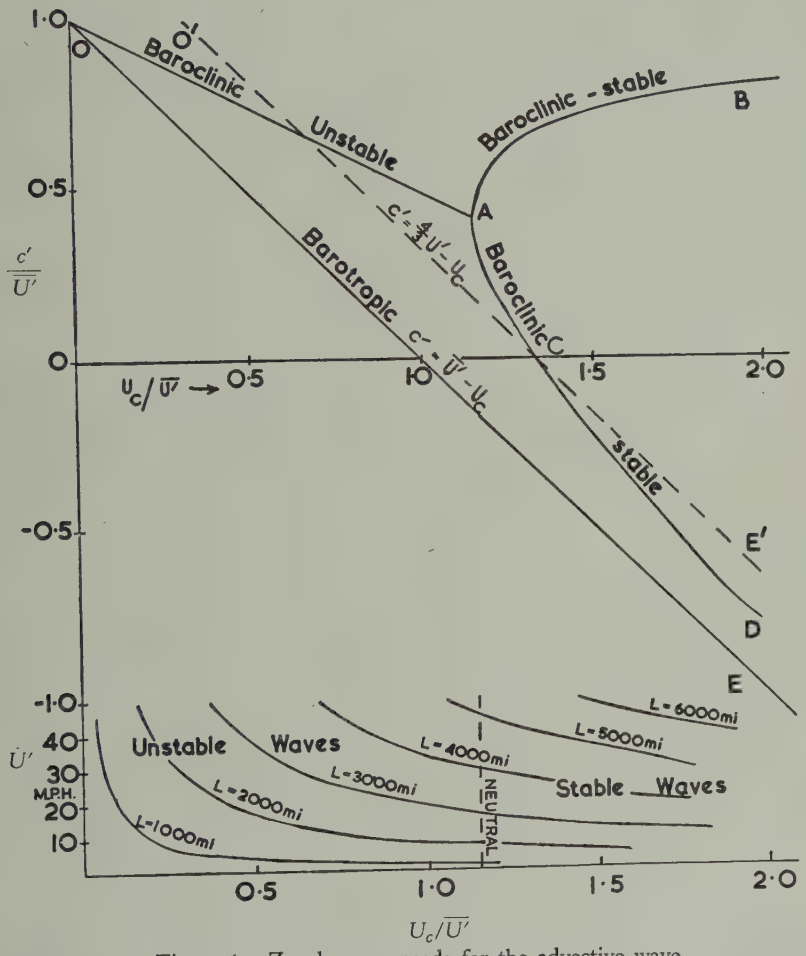

Figure 1. Zonal wave speeds for the advective wave.

The co-ordinates are the non-dimensional quantities $c'/\overline{U'}$ and $U_c/\overline{U'}$ where $c' = c - U_0$, the the wave speed relative to the surface zonal current. $U_c = \beta L^2/4\pi^2$, the Rossby westward component.

OA refers to the shorter unstable waves. A is the critical point where the wave is neutral. DCAB represents the longer waves which are stable. CAB represents progressive antiphase waves, CD retrogressive in-phase waves.

OE represents the barotropic wave speed for the mean zonal velocity. O'E' represents the same but with the zonal velocity taken as 4/3 of the mean value, *i.e.* the value at about 500 mb.

From the lower panel the wave length for latitude 50° corresponding to any point on the upper diagram may be read off, according to the mean thermal wind.

The main features are shown in Fig. 1 where the ordinate is $c'/\overline{U'}$ and the abscissa $U_c/\overline{U'}$. For any given mean zonal thermal wind there is a critical wave length represented by the point A in Fig. 1, such that U_c (the Rossby retrogression component) is equal to the root-mean-square zonal thermal wind or about 1·15 $\overline{U'}$. Shorter waves are unstable developing waves moving with speed c' (relative to the surface zonal wind) represented by the straight line AO. Longer waves are stable but there are two forms with different speeds. The limb AB represents stable, quick-moving, long antiphase waves. If long antiphase waves of this type can exist (warm lows or cold highs) they may move as quickly as much shorter developing waves. This probably explains some occurrences. The section AC represents antiphase waves with speed very rapidly decreasing as the wave length increases to the stationary values at C and then, along the section CD, becoming retrogressive in-phase waves.

In the same figure has been drawn the straight line OE representing the simple barotropic wave speed $c' = \overline{U'} - U_c$. It will be noticed that the formula gives a slower speed everywhere. However, since the pressure-mean thermal wind is usually found at about 600-650 mb whereas the barotropic wave formula may be applied to a convenient 500 mb level the greater U' will compensate for having subtracted U_c instead of $U_c/2$ and the formula in practice would be represented by O'E' parallel to OE. This will pass very nearly through C and thus, if the wind at 500 mb is used in the barotropic formula, the value of the stationary wave length obtained will agree with that derived from our advective model. Agreement will also be obtained for the speed of unstable waves of medium wave length and the barotropic formula may in this way work better in practice than analysis would suggest.

The difficult region near the stationary wave length is specially suggestive. From C, representing the stationary wave, to A, the critical value for stability, there is a decrease of wave length of only about 10 per cent. but an increase of wave speed to near half the mean zonal wind. This probably represents a very real difficulty in synoptic experience ; the sudden stagnation of a long wave which slips into a stationary mode and the equally sudden moving away of a trough or ridge which may have been nearly stationary for days are phenomena presenting much difficulty to the forecaster.

At the foot of Fig. 1 is an additional set of curves giving the wave length in latitude 50° for various mean thermal winds corresponding with the values of $U_c/\overline{U'}$.

4. The growth rate

The growth rate of the unstable waves may be discussed by considering the real exponential factor :

$$\mu c_i = \pi(\overline{U'^2} - U_c^2)^{1/2}/L$$

For critical wave length the growth rate is of course zero but as the wave length is decreased U_c^2 decreases with the fourth power of wave length and the growth rate rapidly approximates to

$$\pi\,\overline{U'^2}^{1/2}/L \simeq 3{\cdot}5\overline{U'}/L$$

The exponential growth rate is therefore effectively proportional to the mean thermal wind velocity and inversely to the wave length. The order of magnitude is indicated by taking one typical value, say L = 5000 km, and $\overline{U'} = 50$ km hr^{-1} when $\mu c_i = 0{\cdot}8$ per day. The amplitude is therefore roughly doubled in 24 hr. The value is not unreasonable.

5. Conclusions

It is not to be expected that such an artificial model will tell us much of theoretical interest. However it may be inferred that there is a real instability in baroclinic zonal flow set up simply by the hydrostatic pressure changes associated with advection of temperature. This instability is present for all wave lengths except the longest, when the variation of Coriolis parameter with latitude stabilises the waves. This type of instability is identical in physical process with the " development " of our earlier theory. It leads impartially to anticyclones and depressions. In the author's opinion this type of instability or " self-developing process " is the one which is fundamental and normal in atmospheric circulations on the synoptic scale.

The type of instability discussed supervenes when the wave length is only a little shorter than the " stationary wave length." Thus there is no clear difference in dimensions between the long stable waves and the developing unstable waves—they are continuous.

The theory suggests $\overline{U'} - U_c/2$ for the speed of the unstable waves.

The theory gives two possible types of long stable wave. The one is rapidly moving with pressure and thermal waves in antiphase and with a relatively pronounced surface-pressure wave, the other is slow moving and either in phase or with only a relatively weak surface-pressure wave.

In the neighbourhood of the critical wave length (near the stationary wave length) the wave speed changes very rapidly with wave length.

The speed of these idealised waves is nowhere given by the " barotropic formula." It is hardly likely therefore that the flow pattern at any level of the real atmosphere can be inferred from barotropic considerations. While the choice of 500 mb may allow a rough overall approximation with barotropic flow patterns there seems little promise in any system which does not distinguish between stable and unstable waves.

It is again emphasized that the author does not expect to find the features of the simple model based on the very much over-simplified dynamics to be found faithfully reproduced in nature. He is looking at the problem as a practical synoptic meteorologist who is seeking the implications of the principles which experience has proved to be helpful in forecasting. Of these the geostrophic approximation is at present forced on the synoptician because he can rarely find a better. The advection of cold and warm air masses is something which certainly takes place and which he regularly estimates in practice. What, he asks, are the dynamical implications of these concepts? Do they when applied to the atmosphere lead to the sort of developments which take place, to long slowly-changing wave-like features, to shorter self-developing systems? Do they suggest that the evolution of the flow patterns in the baroclinic atmosphere is broadly controlled by equivalent barotropic processes and does experience suggest that something more than his readily available approxi-

mations are essential to the understanding and prediction of the main processes? It is believed that this simple study does seem to account satisfactorily for many observed facts and encourages the belief that the broad features of the unstable westerlies are due to an essential instability when warm and cold masses are advected in a sinusoidal pattern. The non-advective temperature changes are known synoptically to be of great importance and the synoptician will probably look first to these to provide explanations for and guides to the features which simple advection fails to predict. He will certainly not be hopeful of much assistance from even more simplification such as barotropic ideas provide, but will expect to find it in the greater complications of baroclinic and thermodynamic processes.

Acknowledgments

This paper is published with the permission of the Director of the Meteorological Office. The diagram was computed by Mr. A. G. Matthewman.

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Billow clouds

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(Manuscript received 15 November 1950)

SUMMARY

The difficulties of deducing a formula for the dimensions or spacing of cloudlets on the assumption that they are convection cells or unstable waves are discussed. The formula

$$\text{wavelength} = 2{\cdot}6938\, h$$

where h is the depth of the statically unstable layer is then deduced for billow clouds of a particular type, on the assumption that they consist of parallel rows of rotating masses of air moving with the mean wind of a shearing layer and sandwiched between stable wave motions above and below.

1. Convection cells

The work of Walker (1931), Phillips and Walker (1932), Durst (1932), and others offers a convincing demonstration that cellular and billow-cloud formations are evidence of a convection-cell motion. Laboratory experiments reproduce many of the patterns so exactly that one feels bound to conclude that the physical processes in the cloud and in the laboratory are the same. By introducing shearing motion, rolls across and along the shear may be obtained, and the cell size seems uniquely determined by the dimensions of the layer in which the overturning is taking place and by the conductivity and viscosity of the air.

As if to confirm this description of the motion Jeffreys (1928) has shown how the cell size can be calculated from the boundary conditions at the bottom and top of the overturning layer, and the conductivity and viscosity. Chandra (1938) gave experimental demonstration that Jeffreys' assumptions were not violated in the laboratory case, so that, even if the problem becomes mathematically intractable when shear is introduced, we can take laboratory experiments as an indication of what will happen. One concludes on this basis that billow clouds are convection cells whose configuration is influenced by shearing motion.

The difficulty of obtaining the regular formations of the cells in the laboratory is seldom emphasised. Considerable care must be taken to obtain steady motion, for the transport of heat must be only just enough to produce any motion at all. Too much heat applied at the base or too much cooling applied at the top immediately makes the motion unsteady. It remains cellular but the life of each cell is small, comparable with the time taken for a particle of fluid to traverse the depth of the layer. Large cells appear, only to be squeezed out of existence a moment later by some formerly insignificant neighbour. Jeffreys shows that besides the dimensions and the properties of the boundary the additional assumption that the motion will only just take place is required in order to obtain the cell size. There is no reason why in nature there should be no overheating or overcooling, in fact generally there is. Convection from the ground more nearly resembles bubbles of hot air ascending, the lifetime of an up-current being of the order of the time taken for a particle to travel the depth of the current.

Another obstacle in the way of quantitative confirmation of the convection cell explanation is that the viscosity and conductivity at altocumulus level are probably negligible. Both act as restraints so that more than an infinitesimal flow of heat through the layer is required before cellular motion sets in, and if they vanish Jeffreys' formulae yield no cell size.

Finally, the resemblance between smoke formations in unstable layers in the laboratory or the appearance of dilute aluminium paint and actual cloud formations is to a certain extent fortuitous, because one would expect all the air that had risen more than the minimum height required to produce condensation to contain cloud. The motion at the top of the layer is horizontal so that a complete layer of cloud would be expected though its thickness might vary regularly. In practice there is almost always blue sky to be seen between the cloud lumps. It seems more likely, therefore, that the lumps of cloud are bubbles of hot air that rise once and for all when the layer becomes unstable, and that steady motion never occurs. The regular disposition of the lumps of cloud may be due to shear, but no quantitative comparison can be made with the very carefully produced steady cellular motions of laboratory experiments. On the other hand if a layer of cloud descends in a general subsidence the clear air below will eventually become warmer and convection will occur in which the clear air in between the cloud globules is the air that has ascended in lumps. The globular form of the cloud is no indication that air is ascending in the middle of the cell and descending round the outside. Nor is a reticular cloud necessarily an indication of descending motion in the middle of the cells. In fact there is no reliable evidence as to which way the circulation goes, except when the convection is due to heating from the ground in which event the up-current seems always to be in the middle of the cell ; or is that merely another way of saying that " thermals " consist of lumps of ascending warm air ?

In no case has any criterion or formula been given which could determine the cell size in the atmosphere where viscosity and conductivity are negligible.

2. Helmholtz waves

An alternative explanation is that bands of cloud are evidence of waves developing in a dynamically unstable air-stream. Holmholtz suggested this because, at a discontinuity of velocity, sinusoidal waves in a certain range of wavelengths initially grow exponentially with time. This did not explain how only one wavelength actually appeared and so, according to Sekera (1948), Wegener arbitrarily assumed that the waves travelled with a velocity which was the mean of those above and below the discontinuity. One wavelength was thereby selected ; but the situation of one semi-infinite stream separated by a plane discontinuity from another contains no linear dimension and cannot possibly yield a selected wavelength. It is highly artificial to overcome this by having two discontinuities, one a short distance above the other, for clearly a model can always be constructed to produce the required length ; it is more realistic to replace the discontinuity by a shearing layer. The mathematical difficulties attendant upon this have been described in a previous paper (Scorer 1951), henceforth referred to as I, and from which the motion used here is taken ; one finds that to have waves travelling with some mean velocity,

vertical stability must be absent in some layer. One is thus led to introduce the required linear dimension in the form of the depth of the unstable layer, but immediately ordinary wave theory fails because vertical overturning occurs and one is back at convection in a shearing layer!

3. Single row of vortices

The difficulties in explaining any cloud phenomenon in terms of dynamically unstable waves have been discussed in I. Chief among these is explaining the ready appearance of only one wavelength. If the waves are supposed to be stable they must first be set up and we assume that their energy is derived from static instability. This also conveniently overcomes the difficulty (Sekera & I) that stable waves in a medium that is statically stable throughout cannot be at rest relative to the air at any level, and the wave velocity is the velocity of the wind somewhere in the statically unstable layer. The ordinary description in terms of small-amplitude sinusoidal waves now no longer applies, and in a shearing layer no mathematical description of convection cells has been given. Indeed a convection cell is not really what is required because, as already pointed out, the unstable layer probably overturns once only whereas a convection cell is a mechanism for the continuous transport of heat. If, therefore, we can find a kind of flow which can give wavy motion outside the layer that is not statically stable and a rotational motion within the layer which can be started by a slight static instability and has no tendency to die away subsequently, then we have reasonable hope that it is a description of some motions that can be observed.

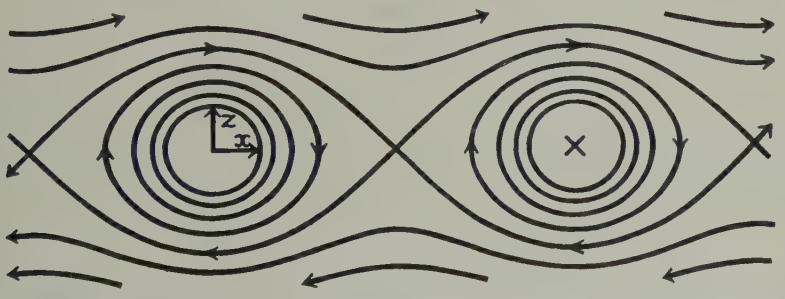

Figure 1. Streamlines for a row of long parallel vortices are shown. In suitable conditions of humidity cloud may appear in parts of the closed circulations. The streamlines at a greater distance above and below are more nearly sinusoidal.

Kelvin had suggested (Taylor 1931) that at the level where stable waves were at rest relative to the fluid they assume the form of a single row of vortices (Fig. 1). The dynamical equations for a statically neutral atmosphere are the same as for a uniform fluid and the complex velocity potential for this motion is (Lamb 1932, Art. 156*)

* The co-ordinate system used by Lamb has y in the vertical. Also $-\kappa/4\pi$ is written for A and $2\pi/a$ for k, and the direction of the circulation is reversed.

$$\phi + i\psi = -2iA \log \sin \tfrac{1}{2} k(x + iz) \quad . \quad . \quad . \quad . \quad (1)$$

The horizontal and vertical velocities are

$$U + u = Ak \frac{\sinh kz}{\cosh kz - \cos kx} \quad . \quad . \quad . \quad . \quad (2)$$

$$\to Ak \operatorname{sgn} z \{1 + \exp(-k|z|) \cos kx\} \quad \text{for large } |z|,$$

$$\ldots \operatorname{sgn} z = \mp 1 \text{ according as } z \lesseqgtr 0$$

$$w = Ak \frac{\sin kx}{\cosh kz - \cos kx} \quad . \quad . \quad . \quad . \quad (3)$$

$$\to Ak \exp(-k|z|) \sin kx \quad \text{for large } |z|.$$

Here the horizontal velocity is denoted by $U + u$; u is the part dependent upon x, and U is the part that transports the fluid. u may thus be regarded as the disturbance of the horizontal velocity. As z increases u becomes more nearly proportional to cos kx and small in magnitude compared with U, and so it is possible to join this flow to the flow described by the ' perturbation ' equations in which we suppose that u depends upon x like e^{ikx} (real part).

The equation governing the variation of w with z for wavelengths less than about $\frac{1}{3}$km, even in a stably stratified atmosphere, is (I, Eq. (22) with $k^2 \gg g\beta/U^2$, and U''/U)

$$w'' = k^2 w \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Terms relating to the static stability are absent because it is a force small in comparison with that due to the dynamical disturbance of pressure for these small wavelengths. If the amplitude dies away exponentially upwards and downwards (*i.e.* rigid boundaries are at a great distance) then

$$w \propto \exp(\mp kz) \text{ according as } z \lesseqgtr 0 \quad . \quad . \quad (5)$$

The disturbance of horizontal velocity (*i.e.* that part depending upon x) is (I, Eq. (20), with $k \gg g/c^2$)

$$u = -\frac{1}{ik} \frac{\partial}{\partial z} w = \pm iw \text{ for } z \lesseqgtr 0 \quad . \quad . \quad . \quad (6)$$

The real part only of this equation is of interest. The appearance of i means that u and w are exactly out of phase. This is precisely the relationship between u and w derived from (2) & (3) when z is large enough for tanhkz to be set equal to unity. The régime of flow in Fig. 1 may therefore be joined on to the solution (5) of (4) at any value of z. Furthermore the static stability does not occur in (4) and so its presence in the air will not disqualify (2) and (3) as a possible motion of the air. It must be absent in the closed circulations in order that they can be started by a slight static instability. Nor is it necessary for U to be constant and equal to Ak sgn z for large $|z|$ for the waves can die away exponentially in any velocity profile provided U does not decrease again to zero at any level. The whole configuration is advected in the general air stream.

The streamlines derived from (1) and used to draw Fig. 1 are

$$z = \frac{1}{k}\cosh^{-1}\{\exp(\psi/A) + \cos kx\} \quad . \quad . \quad . \quad (7)$$

The streamline enclosing the circulations has $\psi = A \log 2$. $\psi = -\infty$ is the centre of the circulations. It is not necessary that the motion in the centres of the circulations should be irrotational since the motion was initiated by static instability and so no difficulty arises if the centre rotates almost as a solid mass. In any case it is not necessary to suppose large angular velocities near the centres.

The line $\psi = A \log 2$ encloses all the air of the originally horizontally-moving unstable layer. If this had depth h then

$$\pi h/2k = \int_0^{\pi/k} z_0 dx \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where

$$z_0 = \frac{1}{k}\cosh^{-1}\{2 + \cos kx\} \quad . \quad . \quad . \quad (9)$$

whence $\frac{2\pi}{k}$ = wavelength $= \pi^2 h \div \int_0^{\pi} \cosh^{-1}(2 + \cos\xi)\, d\xi$

$$= 2{\cdot}6938\, h \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Examples of the kind of billow cloud of which the foregoing is thought to be a fair description are illustrated in Figs. 2, 3, and 4. The analysis refers only to two-dimensional motion and an infinite row of vortices but this is not a serious objection since the formations are in any case short-lived.

Lamb discusses the stability of these configurations of motion and finds that if the central line containing the vortices is distorted sinusoidally the distortion will grow initially like $\exp \lambda t$ where

$$\lambda = \frac{Ak^2}{16\pi^2}\phi(2\pi - \phi)$$

whose maximum value, at $\phi = \pi$, is $\frac{Ak^2}{16} = \frac{Uk}{16}$.

For the typical values $U = 5 \times 10^2$ and $k = 2\pi \times 10^{-4}$, $\lambda = 1{\cdot}8 \times 10^{-2}$ and this extreme disturbance would double its amplitude in about a minute. This is not an unreasonable figure, for these configurations do not generally last longer than about ten minutes, and often vanish in two or three minutes.

The manner in which the system breaks up is not readily described, but it seems likely that any cell that was not of the correct shape would find itself being stretched out over the neighbouring cells until it disappeared. Each cell would be drawn out from its compact form into a thin wavy layer extending above or below its neighbours, and the circulations would vanish.

On Plate II, Fig. 5 shows cloud forms derived from a convection motion proper. The chief difference between this and Figs. 2-4 is that the motion is more obviously cellular and not wavelike. In Fig. 2 the cloud is separated into rolls which seem to be centred on the vortices, while in Fig. 3 the cloud appears to be in the upper part of the circulations only. The wavy configurations are usually more transitory than the cellular ones. An extensive area of billows may be seen in Fig. 4 below the left-hand part of the lenticular mass.

4. Conclusion

The reason for offering this picture of the motion of billow clouds is that differs from convection-cell and unstable-wave explanations in the following :

(a) The cloud rolls are well separated and so no difficulty arises in explainin the blue sky in between them.

(b) The circulations can persist, once they have been set up, with no furthe transport of heat.

(c) They are initiated by static instability, and once set up they die away onl slowly, having become stable waves.

(d) The velocity of the cloud rolls is a mean value of the originally unstable shearin layer.

(e) Since no dynamic instability is involved a single wavelength is readily selected

It is beyond the scope of this paper to discuss how the static instability arises but if it could occur in clear air this type of motion might occur if shear was present This might explain the exceptional regularity of bumps sometimes found by aircraf in clear air.

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The size of cloud droplets in layer-type cloud

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SUMMARY

The rate of growth of cloud droplets round a salt nucleus is examined and it is shown that, with uclei of the size quoted in published measurements, the drop radius achieved in the period before the oplet is carried out of the cloud layer by turbulent diffusion is in reasonable accord with published easurements of the mean drop radius. It is also shown that the maximum drop radius reached when e droplet leaves the cloud owing to fall under gravity is of the same order as the largest cloud-drop easurements. The effect of temperature is considered.

1. INTRODUCTION

Most of the measurements of the size of cloud particles show that these particles ave diameters within the range 5 to 40 μ. The present note is an attempt to explain hy this should be so. Cloud particles form by condensation of water vapour on a nucleus and condensation is essentially a time process. Several authors have omputed the time required for the formation — by condensation — of drops of arious sizes but as far as the present writer is aware only Howell (1949) has taken ccount of the properties of the nucleus.

2. VAPOUR PRESSURE OVER A DROP OF SALT SOLUTION

Wright (1936) has given the equation for vapour pressure over a drop of salt olution in the following form

$$\frac{p'}{p} = \exp\left(\frac{2TM}{a\rho_1 R\theta}\right) - \frac{750 m_1 G}{\rho_1 \pi \beta a^3} \qquad (1)$$

here

p' = vapour pressure over the drop.
p = saturation vapour pressure over a plane water surface at the same temperature as the drop.
a = drop radius.
ρ_1 = drop density.
T = surface tension (74 dynes per cm for water).
M = molecular weight of water (18).
R = gas constant per gram molecule ($8{\cdot}314 \times 10^7$ erg deg^{-1} mol^{-1}).
θ = absolute temperature.
m_1 = mass of solute in drop.
β = molecular weight of solute (58 for NaCl).
G = the proportionality factor in the relation between the fractional lowering of vapour pressure and the number of gram molecules of solute per 1,000 g of water and has a value of 0·04 for NaCl.

It has been pointed out by Simpson (1941) that the second term on the right-hand side of Eq. (1) contains a small error which is significant only at high salt concentrations. The approximate form of the equation given above is convenient for our present purpose and, since we shall be primarily concerned with dilute solutions, will be retained here. For the same reason we shall also assume $\rho_1 = 1$.

As Wright mentions, we easily see from Eq. (1) that there is a value of a which makes p' a maximum. This critical value of a is given by

$$P \exp(P/2a) = \frac{6Q}{a^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where P has been written for $4TM/\rho_1 R\theta$ and Q for $750m_1 G/\pi\beta$. Evaluating P and Q we find

$$P = 6{\cdot}408 \times 10^{-5}/\theta \qquad Q = 0{\cdot}165m_1$$

Thus P is of the order 10^{-7} cm whereas $2a$ is at least 10^{-5} cm and $P/2a$ is small. We may accordingly replace $\exp(P/2a)$ by unity and so get

$$a^2 = 6Q/P \quad . \quad . \quad . \quad . \quad (3)$$

The work of Kohler (1936), Wright (1940) and of Houghton and Radford (1938) indicates that the mass of the salt nucleus lies between 10^{-14} and 10^{-12} g. We shall accordingly consider the six cases given by $\theta = 273$°A and 293°A and $m_1 = 10^{-14}$, 10^{-13} and 10^{-12} g. For each case we obtain numerical values for P and Q, substitute in Eq. (3) and solve for a. Having obtained the critical radii we can then substitute in Eq. (1) and so get p'/p. In passing we may note that we may not substitute unity for the exponential term in Eq. (1), though, if this term is expanded in powers of $P/2a$ sufficient accuracy is obtained by neglecting the second and higher powers. The results for the six cases are given in Table I.

TABLE I. Droplet size for maximum vapour pressure

Case	θ (°A)	m_1 (g)	a (μ)	p'/p
1	273	10^{-14}	2·0	1·000382
2	273	10^{-13}	6·5	1·000121
3	273	10^{-12}	20·5	1·000038
4	293	10^{-14}	2·1	1·000343
5	293	10^{-13}	6·7	1·000108
6	293	10^{-12}	21·3	1·000034

We may note here that unless the relative humidity exceeds 100 p'/p the cloud-drop radius will not exceed the value given in Table I. Moreover, since values of m_1 as small as 10^{-14} g have been deduced as the mean size of the nucleus, and since no measurements of cloud-particle radii have shown the mean drop radius as small as 2 μ, it follows that the relative humidity must, on occasions, exceed 100·038 per cent. On the other hand it seems plausible that the amount of supersaturation is unlikely to be of a greater order of magnitude than 0·038 per cent. both because this would lead to nuclei much smaller than 10^{-14} g becoming effective and because, as the humidity rises, the excess vapour is absorbed by the larger nuclei. In the

llowing paragraphs we shall need an estimate of the relative humidity in a cloud. he argument above suggests that 100·05 is a value which is unlikely to be exceeded.

3. Rate of growth of cloud droplets

Condensation on, or evaporation from, a cloud droplet leads to the drop having temperature greater or less, respectively, than the ambient temperature. If the roplet is so small that the relative motion between the air and the droplet can be eglected it can be shown that (*e.g.* Fuchs (1934) and Topley and Whytlaw-Gray 1927)) the temperature difference between the droplet and the air, and the rate of rowth of the droplet, are described by the following equations :

$$\Delta\theta = \frac{\lambda D}{R\theta k}(hp_a - p') \quad . \quad . \quad . \quad . \quad (4)$$

$$a\frac{da}{dt} = \frac{MD}{R\theta}(hp_a - p') \quad . \quad . \quad . \quad . \quad (5)$$

where

$\Delta\theta$ = excess of droplet temperature over ambient temperature (°A).
λ = latent heat of evaporation (cal/g mol).
D = coefficient of diffusion of water vapour in air ($cm^2\ sec^{-1}$).
k = thermal conductivity of air ($cal\ cm^{-1}\ sec^{-1}\ deg^{-1}$).
$100\ h$ = relative humidity of environment (per cent.).
p_a = saturation vapour pressure at air temperature.
p' = vapour pressure over drop (of salt solution).

and other symbols have meanings as before. Inserting numerical values appropriate to temperatures 273°A and 293°A and, for D, to a pressure of 900 mb we get the following results

θ	$\lambda D/R\theta k$	$MD/R\theta$
273°A	$2{\cdot}041 \times 10^{-3}$	$1{\cdot}955 \times 10^{-10}$
293°A	$2{\cdot}009 \times 10^{-3}$	$2{\cdot}078 \times 10^{-10}$

From Eq. (5) we see that

$$t_2 - t_1 = \int_{a_1}^{a_2} \frac{R\theta}{MD}\frac{a}{(hp_a - p')}da \quad . \quad . \quad . \quad (6)$$

where $t_2 - t_1$ is the time required for the drop to grow from radius a_1 to radius a_2. The integration on the right hand side of Eq. (6) depends upon evaluating $(hp_a - p')$. Now if the temperature of the drop, m_1 and a are specified p' can be computed from Eq. (1). Then if h and $\Delta\theta$ are also specified hp_a can be computed and thus the right hand side of Eq. (4) evaluated. Accordingly the right hand side of Eq. (4) was computed for a few selected values of $\Delta\theta$ and the value of $\Delta\theta$ which satisfied that equation thus determined. Having obtained this value for $\Delta\theta$ the value of $(hp_a - p')$ was determined (Eq. (4)) for the specified value of a. The whole process was repeated for various values of a from $0{\cdot}75\mu$ to 50μ for each of the six cases listed in Table I. Having thus evaluated the integrand in Eq. (6) the integral was obtained graphically. The results are given in Table II which contains additional cases (5a, 5b) for different supersaturations discussed below.

TABLE II. TIME OF GROWTH OF DROPLET

Case	1	2	3	4	5	6	5a	5b
θ	273	273	273	293	293	293	293	293
m_1	10^{-14}	10^{-13}	10^{-13}	10^{-14}	10^{-13}	10^{-13}	10^{-13}	10^{-13}
100 h	100·05	100·05	100·05	100·05	100·05	100·05	100·5	100·025
a (μ)	Time (seconds) to grow from radius $a = 0{\cdot}75\mu$.							
1	2·4	0·15	0·013	1·1	0·08	0·0059	0·06	0·08
2	130	7·0	0·61	52	3·4	0·31	1·4	3·6
5	1,000	320	62	410	150	27	18	260
10	2,700	1,800	870	1,200	870	440	80	2,500
15	5,200	4,200	2,900	2,500	2,100	1,500	210	5,400
20	8,500	7,400	5,900	4,100	3,600	3,000	350	9,100
25	12,500	11,500	9,700	6,000	5,600	4,900	540	13,000
30	17,500	16,000	14,500	8,400	7,900	7,300	760	19,000
35	23,000	22,000	20,000	11,000	11,000	10,000	1,000	25,000
50	44,500	43,500	41,500	22,000	22,000	21,000	1,600	49,000

Now a cloud is always in a more or less turbulent state and during this mixing process elements of the cloud are transferred from the middle of the cloud to the boundary and vice versa. One result of this is that although the cloud as a whole may preserve its identity over long periods it is likely that the individual droplets are continually changing as a result of turbulence transferring the droplets to the cloud boundaries and then into the unsaturated air outside the cloud. Thus the life of an individual cloud particle is limited by the turbulent processes. We can form an idea of the order of magnitude of the life of one droplet as follows. Sutton (1932) has shown that the standard deviation of airborne smoke particles from the mean path is given by

$$2\sigma^2 = c^2(ut)^m \quad . \quad . \quad . \quad . \quad (7)$$

where u is the wind velocity, c has a value about 0·08 and m is 1·75 (cgs units). As a first approximation we may apply this formula to cloud droplets. If we put $u = 1{,}000$ cm/sec we can compute t from Eq. (7) for various values of σ, the latter quantity representing half the thickness of the cloud. Then by plotting a against t for any particular case from Table II we can interpolate values of a for the computed average droplet life. Table III shows the average droplet radius computed in this way for three values of σ (half the cloud thickness). Case 5a has been omitted from the table since it is obviously unrealistic.

TABLE III. AVERAGE RADIUS ATTAINED BY CLOUD DROPLETS

σ (m)	t (sec)	Case number 1	2	3	4	5	6	5b
		Radius (μ)						
100	995	5·0	7·7	10·4	8·7	10·5	12·9	7·0
300	3,485	11·7	13·5	16·0	18·2	19·6	21·2	11·8
500	6,265	16·6	18·3	20·5	25·5	26·5	27·9	16·2

It is not to be expected that Eq. 7 will do more than indicate the order of magnitude of the particle life since, amongst other things, it implies a constant degree of turbulence over the cloud depth. However the values in Table III, particularly for temperature of 273°A and for the more shallow cloud, agree well with published measurements. On many occasions the relative humidity in the cloud will probably be less than 100·05 and will result in particle sizes smaller than those indicated in the table. It will be noted that cloud thickness is as important in determining the size of cloud particles in layer-type cloud as either nucleus size or temperature.

The last two columns of Table II have been included to show the effect of the degree of supersaturation. In section 2 it was suggested that the supersaturation is scarcely likely to differ in order of magnitude from 0·05 per cent. so that case a is of academic interest only. The comparison between cases 5 and 5b is interesting. In the early stages of growth of the droplet the growth will be extremely rapid until the radius approaches the critical value. Before this value is reached the rate of growth decreases considerably. We would therefore expect the time of growth to the critical radius to be approximately inversely proportional to the difference between the ambient vapour pressure and the maximum vapour pressure over the droplet (Table I). Now from Table I we see that the critical radius for cases 5 and 5b is $6{\cdot}7\mu$. In Table II we can interpolate approximately the times of growth to a radius $5{\cdot}7\mu$, getting 350 and 1,000 sec for cases 5 and 5b respectively. The ratio $1{,}000/350 = 2{\cdot}9$ and this is approximately equal to $(0{\cdot}05\text{-}0{\cdot}011)/(0{\cdot}025\text{-}0{\cdot}011) = 2{\cdot}8$. Once the drop radius exceeds the critical value the vapour pressure over the drop diminishes and the time of growth to very large radii will be approximately inversely proportional to the supersaturation.

4. Comparison with Howell's results

Howell's (1949) approach to the problem of the size of cloud particles has been rather different from that outlined above. He has assumed that the governing factors in the size attained by the cloud particles are the size and concentration of the nuclei and the rate at which supersaturation increases as a result of the air rising in convection currents. This contrasts with the present treatment in which the size of the particle is based upon the time the particle spends in the cloud and the rate of growth. Perhaps the two treatments may be regarded as complementary, Howell's method being particularly appropriate to convective clouds and the present method to layer-type clouds.

Nevertheless the two methods overlap in some respects and it is of interest to compare, where possible, the present results with those of Howell. The latter first tabulates a quantity which, in effect, represents the rate of growth of a drop for various assumed values of supersaturation, mass of solute, droplet radius and for a temperature of 273°A and a pressure of 800 mb. The physical equations upon which this table is based are fundamentally similar to those employed here. Approximate values for da/dt can be obtained from Table II above by differencing and Table IV has been computed in order to compare values of this quantity as given by Howell and as obtained from Table II above for approximately the same conditions. The divergence between the two sets of results at small values of a arises from the fact that crude differencing in Table II is not sufficiently accurate at small radii.

TABLE IV. VALUES OF da/dt (CM/SEC) FOR $\theta = 273°A$ AND 100·05 PER CENT. RELATIVE HUMIDITY

Howell			Present Treatment		
a (μ)	m_1 (g)	da/dt (cm/sec $\times 10^{-7}$)	a (μ)	m_1 (g)	da/dt (cm/sec $\times 10^{-7}$)
31·6	58 10^{-16}	0·93	30	10^{-14}	0·95
10·0	,,	2·4	10	,,	2·4
3·16	,,	2·6	5	,,	3·1
31·6	58 10^{-15}	0·93	30	10^{-13}	0·95
10·0	,,	2·4	10	,,	2·5
3·16	,,	5·1	5	,,	4·5
31·6	58 10^{-14}	0·94	30	10^{-12}	0·97
10·0	,,	2·7	10	,,	3·5
3·16	,,	28·5	5	,,	9·2

Beyond this point there is no basis for comparison of the two treatments since Howell thereafter assumes that the degree of supersaturation attained in the cloud is just sufficient to ensure that the rate of loss of water vapour from the air owing to condensation is equal to the rate at which it becomes available above the saturation values owing to the rising and cooling of the air. Nevertheless some of Howell's results are very relevant here. He finds that, ' in rapid convection, with a rate of cooling equivalent to about 4 m sec^{-1} lift, and relatively few nuclei ' the supersaturation reaches its peak of 0·36 per cent. within 25 sec after saturation is reached. On the other hand with ' large nuclei and a slow rate of lift . . . . the supersaturation reached a maximum of only 0·056 per cent.' These figures support the choice of 0·05 per cent. for the maximum value of the supersaturation in layer-type cloud where the rate of lift is small.

5. DISTANCE OF FALL DURING GROWTH

We have seen above that the average size of cloud particles is consistent with the size to which they can grow before they are carried out of the cloud by turbulent diffusion. To compute the latter we used the standard deviation of particles from a mean path. Some particles however will remain in the cloud for a shorter period and some for a longer period than the time corresponding to the standard deviation. The particles which remain in the cloud for a longer period continue to grow but are falling under the influence of gravity the whole time. Their path may be somewhat erratic on account of eddies but over a sufficiently long period the distance of fall is obtained as a product of the terminal velocity and the time. This sets an upper limit to the size of the particles, *viz.* the size to which a drop grows if it falls from top to bottom of the cloud under the influence of gravity. The terminal velocity is obtained from Stokes's law

$$v = \frac{2g(\rho_1 - \rho)}{9\mu} a^2 \quad . \quad . \quad . \quad . \quad (8)$$

(where μ is the coefficient of viscosity) for radii up to about 25 microns. For slightly larger radii the error involved in the use of Eq. 8 is not large (Best 1950) and we shall use it for cloud droplets of all sizes. Neglecting ρ in comparison with ρ_1 and substituting numerical values for the other quantities the coefficient of a^2 on the

right hand side is $1{\cdot}27 \times 10^6$ for $\theta = 273°$A and $1{\cdot}20 \times 10^6$ for $\theta = 293°$A. Using these values and the value of a as a function of t from Table II we can integrate v with respect to t graphically. Table V shows the distance fallen while the droplet radius grows from $0{\cdot}75\mu$ to 'a' microns.

TABLE V. FALL DISTANCE WHILE GROWING FROM A RADIUS OF $0{\cdot}75\mu$

Case	1	2	3	4	5	6	5a	5b
a (μ)			Fall distance in centimetres					
1	0·023	0·0015	0·00013	0·011	0·00075	0·00005	0·00056	0·00073
2	4·5	0·24	0·021	1·7	0·11	0·010	0·044	0·12
5	160	60	12	62	27	5·0	3·0	47
			Fall distance in metres					
10	15	13	6·9	6·6	5·6	3·3	0·50	17
15	67	61	49	31	29	23	3·0	74
20	200	190	170	90	88	80	8·5	210
25	460	450	420	210	210	200	20	470
30	930	910	870	430	420	420	40	950
35	1,700	1,700	1,600	790	780	780	74	1,700
50	5,100	5,100	5,100	2,400	2,400	2,400	160	5,400

From Table V we see that, if the cloud thickness is 600 m, the maximum droplet radius is about $26{\cdot}5\mu$ in case 1 and $32{\cdot}5\mu$ in case 4, values which are in reasonable accord with published observations. Reference to Table II however shows that the time required for this fall would be about 3·8 hr in case 1 and 2·7 hr in case 4. The higher drop temperature not only leads to a larger theoretical maximum drop-radius ; the periods required to grow to the maximum size are comparable with the mean life of a cloud particle and in view of this the reduction of the period from 3·8 to 2·7 hr may make a significant difference to the probability that some droplets will reach the maximum size.

Another point brought out by Table V is the fact that the maximum size of the droplet is but little affected by the size of the nucleus. This contrasts with the effect of nucleus mass on mean drop size already noted.

6. DROP TEMPERATURE DURING CONDENSATION

In the computations described in section 3 it was necessary to compute $\Delta\theta$, the excess of drop temperature over ambient temperature, on the basis of Eq. (4). As a matter of interest the values obtained for $\Delta\theta$ are tabulated in Table VI.

It is apparent from Table VI that except at the smaller sizes, *i.e.* for $a > 5\mu$, the excess of drop temperature over ambient temperature is almost independent of the radius. From the present point of view $\Delta\theta$ is to be interpreted in terms of its effect on the vapour-pressure difference between the drop surface and the surrounding air and we see from Eq. (4) that at a given ambient temperature this vapour-pressure difference $(hp_a - p')$ is proportional to $\Delta\theta$. It is clear from the table that in each case $\Delta\theta$ has a minimum value when the radius of the droplet is

TABLE VI. VALUES OF $\Delta\theta$ — EXCESS OF DROP TEMPERATURE OVER AMBIENT TEMPERATURE (°C)

Case	1	2	3	4	5	6	5a	5b
a (μ)								
0·75	0·018	0·25	3·11	0·035	0·467	6·54	0·522	0·464
1·0	0·006	0·103	1·16	0·013	0·191	2·23	0·245	0·188
1·5	0·0013	0·030	0·323	0·0031	0·056	0·60	0·112	0·053
2·0	0·0008	0·013	0·135	0·0020	0·024	0·25	0·077	0·021
5·0	0·0018	0·0026	0·010	0·0036	0·0051	0·019	0·058	0·0020
10·0	0·0025	0·0026	0·0036	0·0046	0·0051	0·0066	0·058	0·0020
15·0	0·0027	0·0028	0·0030	0·0051	0·0051	0·0056	0·058	0·0020
25·0	0·0029	0·0029	0·0030	0·0056	0·0056	0·0056	0·059	0·0025
35·0	0·0030	0·0030	0·0030	0·0056	0·0056	0·0056	0·059	0·0025
50·0	0·0031	0·0031	0·0031	0·0056	0·0056	0·0061	0·059	0·0025

about the critical value given in Table I. The table also demonstrates the physically obvious fact that the mass of the nucleus has very little effect at the larger drop sizes. In fact, except at the smaller sizes, the value of $(hp_a - p')$ will be independent of drop size and will approximate closely to the value appropriate to a drop of pure water.

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The origin of the electric charge on rain

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Summary

The charges on rain, as measured by Simpson and by Hutchinson and Chalmers, during periods when point discharge occurs, can be accounted for quantitatively on reasonable assumptions in terms of the process of ion capture. The results show that the separation of charge must operate at levels down to about 800 m, and the consequences of this are discussed in relation to theories of the process of separation of charge.

1. Introduction

The observational results of Hutchinson and Chalmers (1951) and the somewhat similar results of Simpson (1949) for the electric charge on rain during point discharge provide empirical relations between the charge per drop, the point-discharge current and the size of drop or rate of rainfall.

It is the purpose of the present work to make use of a very simplified picture in order to provide a first attempt at a quantitative theory of the results and to use this to provide some evidence in regard to the possible processes of separation of charge in clouds. The present work is concerned only with the average values of the charges and makes no attempt to account for the deviations from the average, deviations which the results of Hutchinson and Chalmers show to be sometimes sufficient to reverse the sign ; such deviations clearly must be connected with the fact that the picture used is over-simplified, but works for averages by the cancellation of errors. Results in rain without point discharge are not considered.

The discussion in Hutchinson and Chalmers (1951) showed that the general relation between rain charge and point-discharge current might originate in two ways, and gave reasons for the preference of one, namely that the field giving rise to the point-discharge current is first set up, and then the raindrops acquire charge from the ions produced by point discharge. In what follows, it will be assumed that this is the case and that the acquisition of charge occurs according to the theory put forward by Wilson (1929) and worked out in detail by Whipple and Chalmers (1944).

2. General electrical picture

We must assume that, in the clouds, there is some process, which we shall not discuss further now, by which there is a separation of charge between larger and smaller particles, and that gravitation then produces a vertical separation of charge, the charge of one sign becoming concentrated in the top of the cloud, while the charge of opposite sign falls in the larger particles (raindrops, snowflakes or large ice particles). For convenience of description, we shall consider the case in which the falling particles are negatively charged, but it must be realised that the observations show, by their symmetry with regard to sign, that similar arguments would hold with reversed signs.

The negative charge of the falling particles in the base of the cloud gives rise to a negative field below the cloud and, in the circumstances under consideration, this is sufficient to give rise to point discharge from points near the earth's surface from which a stream of positive ions moves upwards. In the base of the cloud, the falling particles are negatively charged but, as they fall, ion capture occurs and the charge on a drop becomes zero and then positive.

The initial process of separation of charge can operate only under certain physical conditions and so we can expect that there will be a definite lower limit to the region in which the process occurs ; let us call the height of this limit x_4. Above x_4, the falling particles gain negative charge by the separation process and they can acquire positive charge from the ions wherever these exist. So, at some level not below x_4, the falling particles have a maximum negative charge. The analysis to be given later shows that the rate of ion capture near x_4 is small, so that the level of maximum charge is close to x_4 and it is possible to assume that the two processes by which a falling particle acquires charge do not overlap but occur one above x_4 and the other below. Considering lines of force with their positive ends on the earth, on the positive rain near the earth or on the positive ions moving upwards, and their negative ends on the falling particles in and below the cloud base we see that somewhere near the base of the cloud there must be a region of zero field and therefore of zero ionic current. If convection currents can be neglected here, this must be the region of maximum negative precipitation current, since elsewhere positive ions contribute to the total vertical current. Thus this level of zero field is also at x_4.

If electrical conditions remain steady, the total vertical-current density must be the same at all levels and therefore equal to the precipitation current at x_4. It may be mentioned that the same current density must also exist above the cloud where it could be carried by ions in the region between the top of the cloud and the ionosphere.

Near the earth's surface, drops and ions carry positive charges and this positive space charge gives an increase of field with height. Higher up the drops are negative and so, at some point above that at which the drops have zero charge, the negative space charge of the drops balances the positive space charge of the ions and the field has a maximum value ($\rho = 0$, $dF/dx = 4\pi\rho = 0$, $F =$ maximum), decreasing at higher levels because of the negative space charge of the drops and ultimately becoming zero at x_4.

The calculations to be given are concerned with deducing the value of x_4 consistent with the observations for all sizes of drop or rates of rainfall.

3. Discussion of assumptions

In order to provide a quantitative treatment of the phenomena, it is necessary to make certain assumptions and approximations which are here set forth.

The most fundamental assumption is probably that, in considering average values, it is possible to take the conditions as steady. In fact, electrical conditions are always changing, but, when averages are considered, it is probably true that the divergences from steady conditions will balance out. Simpson (1949), in discussing

is results, pointed out that the point-discharge current and the rain charge often hange sign simultaneously and suggested that this was an argument against the ain acquiring its charge at a high level from which it would take some time to fall. n answer to this it may be suggested that changes in field at the earth's surface re more probably due, not to a change in the the electrical structure of a portion of he cloud (except, of course, in the case of lightning flashes) but rather to the travel, ver the observer, of the cloud, different portions of which differ electrically ; there hen might be, travelling at approximately the same speed past the observer, the vhole system of cloud, rain charge and field ; and the rain charge and field would hange sign nearly simultaneously even though the rain acquires its charge at a onsiderable height.

The next assumption arises from the results of Hutchinson and Chalmers on he comparison of drop charge and $-3X_0a^2$ (*cf.* Section 4 for nomenclature). The aindrops cannot have acquired their charges by Wilson's process unless the field s considerably greater aloft than at the earth's surface. The discrepancy between he field increase to be expected from the space charge and the absence of such ncrease in alti-electrograph results (Simpson and Scrase 1937) is discussed by halmers (1949). If the results under discussion are to be accounted for in terms f Wilson's process, it must be assumed that fields increase with height according o space-charge theory.

For the purpose of making calculations it is necessary to assume that all the aindrops during a period of rainfall are of the same size. This is certainly not so, ut it is an approximation which cannot be avoided and again, divergences will end to balance out in considering averages. With this assumption, it is possible to use Best's (1943) D_v, R relation and so, from Simpson's (1949) rates of rainfall, calculate the average size of drop, or, from the drop sizes of Hutchinson and Chalmers (1951) calculate the average rate of rainfall.

It is also necessary to assume that all drops falling at one time carry the average charge. In fact, the observations have shown that different drops carry different charges, even of different sign, but it is the aim of the present work to deal with averages, not with deviations from the average.

It is assumed that there is a definite " effective separation of points " for the process of point discharge, relating the point-discharge current density with the current through a single point, and that this separation does not depend on the magnitude of the currents.

Vertical convection currents are neglected in comparison with precipitation currents and ionic currents. This assumption is probably justified in most of the range of heights concerned.

Another assumption is that there is no appreciable combination of point-discharge ions with Aitken nuclei, producing large ions of much lower mobility. The time taken by small ions to travel from the earth up to a level near x_4 in the fields that are later calculated to exist is of the order of a minute ; very near x_4, however, fields and hence ionic velocities are small, so that the chance of combination is there greater. The life of a small ion near the earth's surface is of the order of one minute but higher up, where Aitken nuclei are less plentiful, the life will be longer. Thus the assumption may be nearly justified.

Although different meteorological conditions obtain during different periods of rainfall, giving different heights at which the separation of charge occurs, the use of averages from the observations requires that we assume one definite average height for the process and so one average height for x_4. It is assumed that this is the same for all rates of rainfall or all sizes of drop. When more observations are available, it may become possible to differentiate between different conditions and to find how these affect x_4.

It is assumed that the process of ion capture is by water drops, though ice particles might also be effective (Chalmers 1947).

In addition to assuming steady electrical conditions, we must assume that the equipotential surfaces are horizontal. This is clearly seldom exactly true, particularly when the field is changing sign, but it seems likely that deviations from the assumption will at times be in either direction and so, on average, will balance out.

4. Notation

In the notation which follows the signs as given refer to the case of positively-charged rain in a negative field.

a = radius of drop (cm).
D_v = diameter (mm) of drop of average volume.
V = velocity of drop (cm sec^{-1}).
Q = charge of drop (esu).
x = height above ground (cm).
$-X$ = field or potential gradient (esu) ; negative sign since it is everywhere negative.
N = number of drops cm^{-3}.
n = number of positive ions cm^{-3}.
e = charge on ions (esu).
w = mobility of ion (esu).
J = $newX$ = ionic current density (esu) upwards.
i = NVQ = rain current density (esu) downwards.
I = point-discharge current (esu) through a single point.
d = effective separation of points (cm).
A = $d^2 = I/J_0$ = effective area for one point (cm^2).
R = rate of rainfall (cm sec^{-1}).
R' = $R \times 36{,}000$ = rate of rainfall (mm hr^{-1}).

With suffices :

0, at earth's surface ; 1, where $Q = 3Xa^2$; 2, where $Q = 0$; 3, where $Q = -3Xa^2$; 4, where $X = 0$.

Q_4 is negative and we put $-Q_4 = T$.

It is convenient to use certain substitutions so that results can be obtained for various conditions and not only for single values of the variables. We take the following non-dimensional parameters :

$$F = wX/V \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$M = ne/NT \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$P = Q/T \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$s = 3a^2V/wT \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$y = \frac{4\pi wNTx}{V} = \frac{12\pi a^2Nx}{s} \quad . \quad . \quad . \quad (5)$$

The quantities a, V and N depend on the rate of rainfall considered, w can be considered constant and T depends on the charge given to the drop by the separation process in the cloud.

5. General theory

Whipple and Chalmers (1944) have given a complete theory of the capture of ions by water drops, on the assumption of a uniform field in which the drops are situated and of a current of ions of one or both signs.

The present discussion concerns fields which are not uniform on a large scale but which can certainly be considered uniform over the size of one drop. The ions are of one sign only and the appropriate equations from Whipple and Chalmers apply ; the fact that these equations assumed an independent source of ionic current while in the present application the field and ions are primarily due to the charges on the drops, does not affect the argument, since the motion of ions in the field near a drop depends merely on the ions and the field, and not on the origin of either.

From Whipple and Chalmers the ion-capture theory gives :

$$\text{for } 3Xa^2 > Q > -3Xa^2, \quad \frac{dQ}{dx} = -\frac{\pi new}{3Xa^2V}(3Xa^2 - Q)^2 \quad . \quad . \quad . \quad (6)$$

$$\text{for } Q > 3Xa^2, \quad \frac{dQ}{dx} = 0 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\text{for } Q < -3Xa^2, \quad \frac{dQ}{dx} = -\frac{4\pi new}{V}Q \quad . \quad . \quad . \quad . \quad (8)$$

If electrical conditions are steady and equipotentials horizontal, the total vertical-current density must be the same at all levels and equal to that at x_4, where $X = 0$ and $J = 0$.

$$\text{Thus } NVQ - newX = NVQ_4 = -NVT \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From Poisson's equation :

$$\frac{dX}{dx} = 4\pi(ne + NQ) \quad . \quad . \quad . \quad . \quad (10)$$

Substituting from (1) to (5) in (6) to (10),

$$\text{for } sF > P > -sF, \quad \frac{dP}{dy} = -\frac{M}{4sF}(sF - P)^2 \quad . \quad . \quad . \quad . \quad (11)$$

$$\text{for } P > sF \quad \frac{dP}{dy} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\text{for } P < -sF \qquad \frac{dP}{dy} = MP \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\text{and for all } x < x_4 \qquad MF = 1 + P \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$\frac{dF}{dy} = M + P \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The equations (14) and (15) together with the appropriate one of (11), (12) or (13) form the differential equations which, on solution with the boundary conditions, will give values of P, F and M for any value of y, or, re-substituting from (1) to (5), the values of the drop charge, the field and the ionic density at any height.

For $P > sF$ or $Q > 3Xa^2$, (12), (14) and (15) can be integrated easily. X_0 or F_0, at the earth's surface, is small compared with the corresponding quantities higher up and if we assume that $F_0 = 0$ then the value of y_1 can be obtained for any given P_0 and s as :

$$y_1 = \frac{1}{s} - \frac{1 + P_0}{P_0^2} \log \left(1 + \frac{P_0^2}{s(1+P_0)}\right) \quad . \quad . \quad . \quad (16)$$

This gives the value of y_1 or, from (5), the value of x_1 at which $P = sF$ or $Q = 3Xa^2$, the height at which the field has the value required to give the drop its maximum charge by Wilson's process ; below this height, as $Q > 3Xa^2$, (7) shows that the charge does not alter.

For $sF > P > -sF$, which usually gives the greatest part of the total height concerned, the equations (11), (14) and (15) can only be solved by numerical methods, as will be discussed later.

For $P < -sF$, analytical solutions of the equations (13), (14) and (15) exist, but partly in series form and in practice it is more convenient to use the same numerical methods as for the previous range. As y increases, *i.e.* as one rises from the earth, P decreases to zero and then becomes negative, reaching -1 when $Q = T$ at x_4 ; meanwhile F increases and then decreases, becoming zero at y_4, so that at y_3, below y_4, $P = -sF$.

The physical interpretation of the limits $P = \pm sF$ or $Q = \pm 3Xa^2$ can be obtained from Whipple and Chalmers' analysis. When $Q > 3Xa^2$, the whole drop is positively charged and so no positive ions can be attracted ; when $3Xa^2 > Q > -3Xa^2$, polarisation in the field gives part of the drop a negative charge, and positive ions can reach this part. For $Q < -3Xa^2$, the whole drop is negatively charged, and the normal conductivity formula holds.

When we are close to y_4, F is small and the analysis shows that M becomes small also, so that at y_4 or x_4, where there is zero field, there is also zero space charge of ions.

Actual measurements give the charge Q_0, on the drop at the earth's surface, the point-discharge current I through a single point and either a, the drop radius, or R, the rate of rainfall, and hence from Best, V and N. The mobility w of the ions and the charge e per ion are assumed known. If the effective separation of points, d, is assumed, then J_0 is known and (9) shows that $-NVT = NVQ_0 - J_0$, so that T is obtained. Hence, for any given observations and an assumed value of

, it is possible to obtain the quantities P_0 and s. Calculations can then be made or various values of P_0 and s.

6. Method of calculation

The method of numerical calculation uses equations (11), (14) and (15) in the following way.

If F and P are known for some particular y, it is possible to obtain $F + \delta F$ and $P + \delta P$ for $y + \delta y$. Slightly greater accuracy is obtained by using for δF and δP not the values calculated for y, but the mean of those calculated for y and for $y + \delta y$; The process is repeated for successive steps of δy and it often becomes convenient to alter the magnitude of δy in different parts of the calculation.

Calculations are started for $y = y_1$, when $P = P_0$ (since $dP/dy = 0$ for $y < y_1$), and $F = P_0/s$. The calculations are continued until the stage is reached when $F = 0$ and $P = -1$. when $y = y_4$. The numerical calculations thus give $y_4 - y_1$ and hence y_4 using y_1 from (16).

The table shows a sample portion of one of the calculations, that for $P_0 = 1$, $s = 3$.

y	$\frac{dF}{dy} = M + P$	F δF	$sF - P$	$\frac{-dP}{dy} = \frac{M(sF-P)^3}{4sF}$	P δP	$M = (1+P)/F$ P $M + P$
0·2	2·65	1·093 0·242	3·279 0·907	0·75	·907 ·080	1·74 ·91
			2·372			2·65
0·3	2·20	1·335 ·203	4·005 0·827	0·86	·827 ·089	1·37 ·83
			3.178			2·20
0·4	1·86	1·538 0·170	4·614 0·738	0·92	·738 ·094	1.12 ·74
			3·876			1·86
0·5		1·708			·644	

Fig. 1 shows the complete variation of P, F and M with y for this case ; this means that, for certain particular observational values of Q_0, I and a, and a suitable choice of d such that $P_0 = 1$, $s = 3$, then Q, X and n can be found for any value of x by substituting back from equations (1) to (5) and using Fig. 1. Similar curves can be drawn for other values of P_0 and s.

Such calculations have been carried out for 63 different cases, with P_0 ranging from 0·03 to 10 and s from 0·01 to 100. The results have been plotted as curves relating sy_4 (more convenient than y_4 itself) against P_0 for different values of s ; Fig. 2 shows some of these curves. The value of y_4 for any values of P_0 and s can easily be found by interpolation. These curves can be used for comparison with observation as follows.

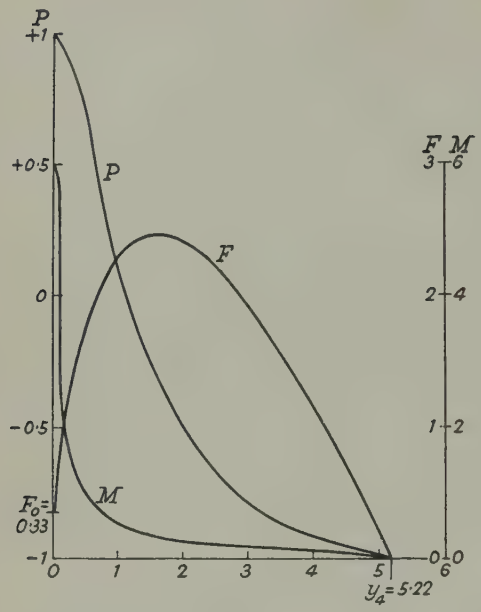

Figure 1.

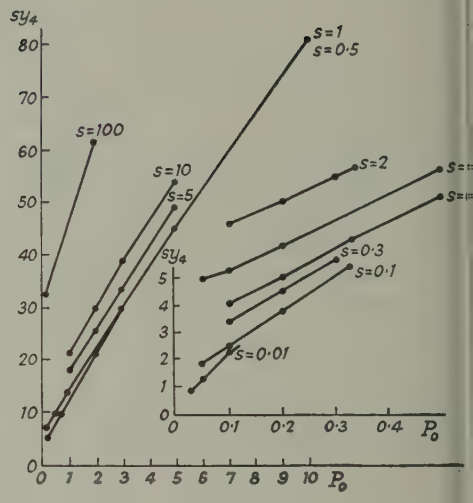

Figure 2.

Simpson's observational results for i and I were grouped together for five different ranges of R' and, after taking into account the different durations of the separate observations, averages were obtained for R', i and I in each case. The corresponding average radius a was found from Best (1943) and of V from Laws (1941). Since $i = NVQ_0$, Q_0 can be found. With any choice of d, T is obtained and then s and P_0 are deduced, taking $w = 450$ ESU ($= 1{\cdot}5$ cm sec^{-1} for 1 v. cm^{-1}). From the $sy_4 - P_0$ curves, y_4 is obtained and thus x_4 from (5). From (9) and (3), $Ai_0/I = P_0/(1+P_0)$, so that Ai_0/I can be obtained in each case also.

For each group of results for one mean R' (and a), it is possible to draw a curve relating x_4 and Ai_0/I, the different points arising from different assumed values of s (and so of d). Using such results from the five separate ranges of R', it is possible to determine what values of x_4 and A give the best fit to the observational averages i_0/I for all 5 ranges.

The procedure with the results of Hutchinson and Chalmers is similar, the only differences being that a is known and R has to be deduced from Best's results, and that observations require Q_0/I which is given by $AQ_0/I = P_0NV/(1+P_0)$. Again we can find the values of x_4 and A which give the best fit to the observational averages.

7. Results of calculations

The results of the calculations show that there is quite a range of values for x_4 and A which give good agreement with the observational curves. By the method of least squares, it is found that the best agreement with Simpson's results occurs with $x_4 = 0{\cdot}7 \times 10^5$ cm, $A = 1{\cdot}77 \times 10^6$ cm^2, while for the results of Hutchinson and Chalmers the best values are $x_4 = 0{\cdot}9 \times 10^5$ cm, $A = 0{\cdot}40 \times 10^6$ cm^2. These are shown as the full lines in Figs. 3 and 4.

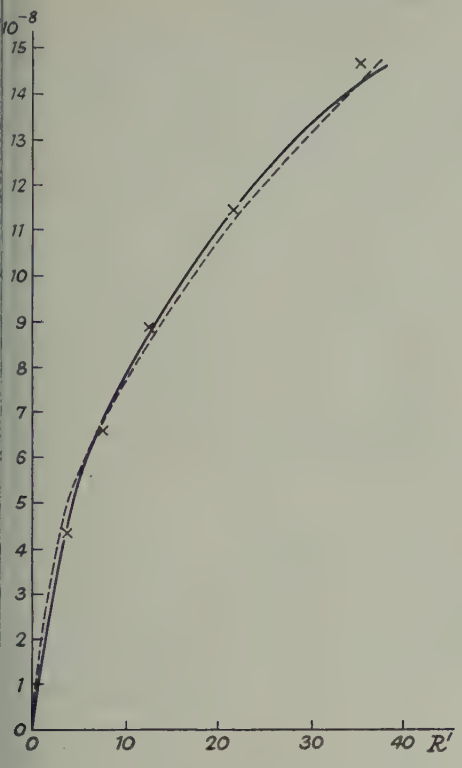

Figure 3.

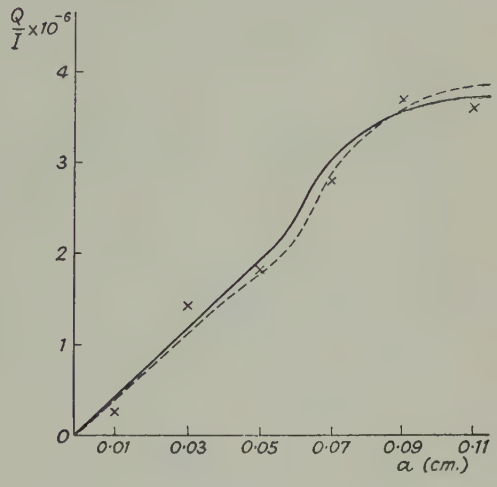

Figure 4.

There seems no reason why x_4 should differ for the observations at Kew and Durham, and a mean value of $0{\cdot}8 \times 10^5$ cm. is accordingly chosen for x_4. The corresponding values for A are $2{\cdot}12 \times 10^6$ cm^2 and $0{\cdot}37 \times 10^6$ cm^2. These results are shown as the dotted lines in Figs. 3 and 4 and it can be seen that they are little inferior to the full lines.

The value of $2{\cdot}12 \times 10^6$ cm^2 for A for Kew means an effective separation of discharging points of 14·5 m, to be compared with the original estimate of 25 m (Whipple and Scrase (1936)). For Durham the separation is 6·1 m.

8. Discussion of results

With the results shown by the dotted lines, and making the assumptions discussed earlier, the electric charges on raindrops during periods of point discharge can be accounted for on the average if the falling particles acquire charge by some process operating at heights down to 800 m and then set up a field in which ion capture occurs. The corresponding effective separations of discharging points are not unreasonable.

If the process of separation of charge operates down to 800 m on the average, then this may be used to give some indication of the nature of the separation process.

It may first be emphasised that our assumptions imply that it is the precipitation particles, rather than the body of the cloud, which are charged by the separation process, the charge of opposite sign rising as smaller particles, perhaps cloud droplets, or ions. If the process of separation of charge is associated with ice particles as has been suggested by Simpson and Scrase (1937), Findeisen (1943) and others, then we must expect that the precipitation particles would on the average remain solid down to the level x_4, the separation process operating thus far but not lower.

Murray (1950) has published results for the mean freezing-point level over the British Isles, and from these one gets a mean value for Durham and Kew of around 1,800 m ; this, however is a general mean for all types of weather and the mean for periods of rainfall would certainly be very much lower, especially considering only those occasions when point discharge occurred and when, therefore, there was considerable separation of charge, often in cold-front conditions. Without a detailed analysis it is difficult to estimate the average freezing level under such conditions, but it seems not impossible that it might be reduced to 1,200 m or even 1,000 m.

The actual level of melting is lower than the freezing-point level, and the magnitude of this difference is, on the average found from the " bright band " phenomenon by Hooper and Kippax (1950) to be about 100 m ; for the large drops concerned in the present discussion the difference is certainly greater.

While the present results give no certain indication that the process of charge separation is to be associated with ice particles, this is not to be entirely ruled out.

The symmetry of the observations in regard to sign seems to suggest that the process of separation of charge can operate either way round and at the same level. The observations discussed here would not support a distinction of level between two processes (or opposite signs of the same process) of separation of charge, such as is indicated for thunderstorms by Simpson and Scrase (1937). More usually, the process of separation of charge gives a negative charge on the falling particles at x_4, hence a negative field and positively charged rain at the ground ; but when the process is reversed, x_4 is nearly the same.

If it can be considered that the process of separation of charge occurs with ice particles, then the theory, based on observational evidence, that seems best suited to the present conclusions may be that of W. and E. Findeisen (1943), who found observationally that charged splinters are formed from ice particles when they grow by sublimation or when they evaporate ; normally the splinters are negatively charged when the ice particles are growing, and positively when they are evaporating, the actual particles getting the charge of opposite sign. They also found that, in strong fields, the splinters take the sign of the vertical field, thus giving rise to an increase of the existing field. Thus the more usual negative field is due to ice particles evaporating in the base of the cloud and in the less usual case of positive field, the particles are growing by sublimation. If this is so, we might well expect that larger drops will occur in conditions of positive field and negative rain charge ; Hutchinson and Chalmers found that the correlation of a and Q gave a hint that larger drops tend to be negative.

The fact that snow shows considerable charge must indicate that there is separation of charge occurring in the absence of liquid water.

The present conclusions concern only average results ; individual drops differ from the average for many reasons, among which may be different histories as ice particles and the unsteadiness of conditions. Calculations to account for deviations from the average do not appear to be feasible by the present methods.

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On the diurnal variation of precipitation, particularly over central U.S.A., and its relation to large-scale orographic circulation systems

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SUMMARY

The diurnal variation of precipitation is discussed and attention is drawn to the high frequency of night-time rains over central U.S.A. A new theory for explaining this nocturnal maximum and the relatively low activity of precipitation in daytime is proposed by suggesting that cooling and heating processes during the night and the day set up a large-scale circulation system east of the Rocky Mountains. The existence of this circulation system is proved by computation of the changes which occur in the convergence-divergence field at 4,000, 10,000 and 18,000 ft in the free atmosphere.

1. INTRODUCTION

Synopticians and climatologists have paid considerable attention to the diurnal variation of precipitation as it presents itself over different parts of the globe. In many textbooks on synoptic meteorology the influence of the diurnal changes in the vertical stability of the atmosphere on the formation of convective clouds and their precipitation over land and sea surfaces has been described, and every synoptician knows that in sufficiently moist air, unstable enough to allow convection, the maximum of shower activity over land is found in the afternoon, whereas over sea the later part of the night shows the greatest rainfall from showers. In climatological textbooks distinction has been made between the maritime type and the continental type of the diurnal variation of precipitation both of which are also considered to be related to the changes in stability of the lower part of the troposphere.

It is, however, well known that there are several parts of the earth where over land the above-mentioned idealized picture is blurred, especially in coastal and mountainous areas. Braak (1940) for instance discusses the double precipitation-maximum in coastal regions of the Netherlands and of Indonesia, as well as the deviations from the normal diurnal variation over and near mountains. The two maxima in coastal regions can easily be explained as a combination of land influences and sea influences, the latter being a result of showers during the night (by on-shore winds) transported from the sea. In mountainous areas katabatic winds during the night descending from the slopes may cause local convergences as a result of which clouds and nocturnal rains in the valleys are observed. Braak mentions the Paloe valley (Celebes), as an example of a "maritime rainfall type" over land. There, during day-time, the mountains are always in the clouds while in the valley cloudiness is substantially less whereas, during the night, the mountains are free from cloud and the valley is overcast, sometimes with rains.

American meteorologists have brought to light that there exists an extensive area over the continental part of the United States where a marked nocturnal maximum

* Work done at the University of Chicago under the sponsorship of the Office of Naval Research, U.S.A.

f rainfall is present. Apart from several short studies on the diurnal variation f rainfall at a number of stations in U.S.A., American literature contains an interest-ng paper published by Kincer (1916) in which a map is reproduced showing that n summer over the central plains of U.S.A. a large area is present where 50 per cent. r more (in Nebraska even over 65 per cent.) of the precipitation falls during the ight (8 p.m. - 8 a.m.). Moreover many nocturnal rains fall in showery precipita-ion. This can be derived from the valuable information contained in the *Hydro-neteorological Report* No. 5 (1947).

Fig. 1, reproduced from this report, gives the percentages of thunderstorm occurrences for the quarter-days 00-06, 06-12, 12-18 and 18-24 hr local ime. It will be seen that from 18-24 hr a zone of considerable thunder-storm activity stretches from north to south over central U.S.A., and that rom 00-06 hr important thunderstorm activity still exists over the Midwest. Striking is the minimum of thunderstorm activity over the central plains during the period 12-18 hr. It is certainly much more difficult to make investigations on the contribution of fronts to night-time rains so that up to now no climatological data of the diurnal variation of frontal precipitation over the Midwest are available. However, Dexter (1944), in a discussion on the diurnal variation of warm-front precipitation and thunderstorms, gives some examples of the widening of a frontal rain belt during the night over an area where also a marked maximum of thunder-storm activity is found. This might indicate that the factor contributing to high night-time thunderstorm activity is also present in situations where fronts cover the area in question.

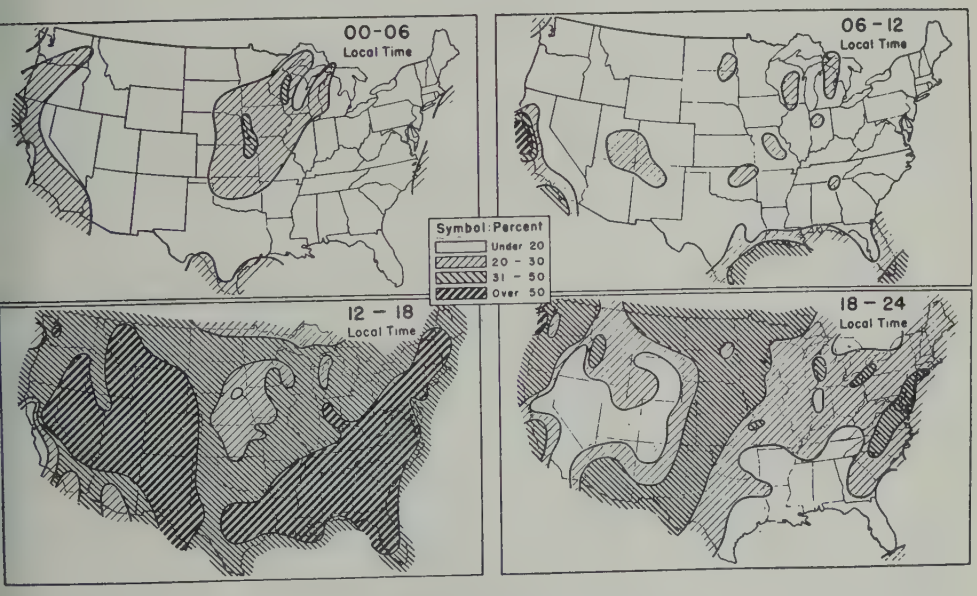

Figure 1. Percentages of summer thunderstorm occurrence per quarter-day (from *Hydro-meteorological Report*, no. 5).

2. Previous discussions of night-time rainfall and night-time thunderstorms over continents

Various attempts have been made to explain high night-time rainfall and night-time thunderstorms occurring over some continental areas. A short description of the ideas of different authors will be given. As the reader will see, most of the suggestions do not provide an explanation of the precipitation *patterns* as they are found over U.S.A.

(a) Several authors (see for instance Hann 1926) have explained the night-time maxima of rainfall as a direct result of radiational cooling. Hann seems to presume that this cooling causes condensation, which will be measured as precipitation. In the light of our present knowledge of the formation of rain, one can not expect that the formation of fog and stratus can have great influence on the diurnal variation of rainfall.

(b) In the *Hydrometeorological Report* (1947) the suggestion is made that the higher relative humidities of the night might diminish the evaporation of the falling precipitation. The objection which could be made to this idea is that the rate of evaporation of precipitation elements is not dependent on the relative humidity of the air, but on the difference in vapour pressure over the cool precipitation element and the air, which difference is likely to have no great diurnal variation.

(c) Dexter (1944), seemingly referring to a study of Goldie (1931) on the diurnal variation of warm-front rain over the British Isles, which shows a maximum in the latter part of the night or the early morning, explains this maximum as a result of the diurnal variation of the potential instability. Warm-sector air, becoming potentially more unstable during the day, slides upward along the warm-front surface and reaches its condensation level during the night, thus giving more intense precipitation (also in greater quantities) during this time than during the day when air stabilized during the night climbs the warm-front surface.

(d) It has been suggested by Hewson (1937) that the greater night-time precipitation might be caused by a greater instability in the cloud layers, which will develop as a result of the radiational cooling of the top and radiational warming of the bottom layers of the clouds. Dexter (1944) and Means (1944) have criticized this idea ; Means, for instance, expresses the opinion that radiational cooling of the top would lead to the development or intensification of a stable layer or inversion above the top, which would again suppress vertical motions.

(e) Another theory on the occurrence of night-time thunderstorms is based on the idea that advection of cold air aloft might destabilize the air to such an extent that convective showers could occur even during the night. Means (1944) who has published the most recent ideas on the development of Midwest thunderstorms is of the opinion that the advection of *warm* air in the lower layers is an important factor in the development of night-time thunderstorms. Though this advection has a stabilizing effect on the lower layers, it has a destabilizing influence on the higher layers. Means studies the *mean* isobars and isotherms over U.S.A. and he points out that their orientation is such that warm-air advection can be expected at levels of 5,000 and 10,000 ft. Referring to data on the diurnal variation of winds in the free atmosphere compiled by Wagner (1939) Means points out that the advection of warm air is greater during the night than during the day which could account

for the diurnal variation of thunderstorm activity. The so-called advective effects computed by Means show a geographical distribution resembling that of the frequency of night-time thunderstorms. However, Means's reasoning is not convincing; if one considers isobars and isotherms on *mean* charts their intersection only shows that important factors other than advection, as vertical motion and non-adiabatic processes, must have been present. Since at present it is not possible to estimate the value of the adiabatic and non-adiabatic factors, it seems rather dangerous to draw conclusions on the influences of advection on a particular day or a particular night from *mean* patterns.

3. The influence of the topography on the diurnal variation of precipitation over the eastern part of U.S.A.

From the above it will be clear that other causes have to be sought for the diurnal variation of the precipitation and the occurrence of thunderstorms over the U.S.A. east of the Rocky Mountains. Further, it is not only of importance that the high night-time precipitation and thunderstorm activity over the Midwest be explained, it is also necessary to understand the relatively low rainfall and thunderstorm figures during the day. Since the heating during day-time is a greater destabilizing factor than many other factors brought into discussion by the authors mentioned in section 2, then it is clear that during day-time there must be a process suppressing the general precipitation activity, particularly when this precipitation tends to fall in the form of showers and thunderstorms.

A process analogous to that which causes the high night-time precipitation and convective activity must have a counteracting influence during day-time. It is obvious that large-scale periodically ascending and descending motions must be present in the area discussed. Ascending motions during the night will lead to the formation of clouds when the moisture content of the air is high enough; these clouds are layer clouds or convective clouds with high base. Such ascending motion will also cause a general destabilization of the layers above the cooled surface layer, which is particularly important in cases of potential instability. Subsidence during the day will stabilize the layer and restrain the vertical development of convective clouds.

The following mechanism is suggested as causing ascending and descending motions during night and day.

In Fig. 2 an idealized orographical situation is sketched, in which a broad plain (several hundred miles) is bordered on both sides by more or less slightly sloping mountain areas. Plain and mountains are covered by originally barotropic air, the isobaric surfaces of which are indicated by thin horizontal lines. Now

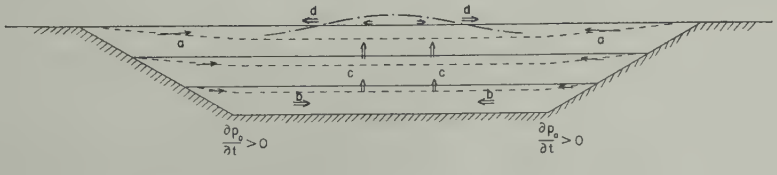

Figure 2. Idealized scheme of the circulation developing during the night.

when nocturnal cooling sets in the isobaric surfaces in the higher parts of the free atmosphere will sink, and by approximately the same amount in each level except towards the mountain slopes where the cooled layer under the isobaric surface is thinner than over the plain. It seems safe to assume that as a rule even the influence of a more intensive cooling in the slope regions will not be great enough to make the lowering of the isobaric surface greater there than in the plains.

From this schematic picture we see that especially in the higher layers there should be a transport of air to the plains ; this transport of air will not in the first instance extend into the plain area itself since horizontal pressure gradients do not develop there initially ; it may however be expected that the transport of air will gradually extend also to adjacent plain areas. The plainward transport of air will cause a rise of pressure, which will have its highest value at the surface of the earth, especially in the regions near the feet of the slopes, indicated by $\partial p_0/\partial t > 0$. As a result of this pressure rise, which will gradually extend further into the plain, a low-level convergence will develop in the plain itself, with consequent ascending motion over the plain. There will be a rising of the isobaric surfaces over the plain with a diverging motion in the upper air and this will be enhanced if condensation occurs and latent heat is liberated. During day-time the opposite processes will occur. Then the strong rise of isobaric surfaces over the plain must cause a falling pressure at the feet of the slopes and a divergence in the lower layer of the plain, with descending motion and convergence in the higher layers. Thus over the plain area there will be alternating ascent and descent (ascent during the night, descent during the day). The circulation system thus formed has much larger dimensions than the small-scale systems of heated air rising along steep mountains during the day or cooled air katabatically descending during the night, (compare Wagner 1932). It can be compared with that described for the first time by Saigey (see Hann 1926) and used by Hann as an explanation for the periodic winds occurring in the Himalayas. In our idealized scheme of the circulation (Fig. 2) it is not essential to have a symmetric situation ; the same effect would be present if we had only *one* sloping mountain system bordering a plain. In U.S.A. the plain area where the effects discussed will occur is bordered by the Rocky Mountains on the west, and by the Apalachians on the east.

In a study of upper-air winds at eleven stations in the United States over two successive summers, Wagner (1939) concludes that there are three diurnal circulation systems superimposed on the general wind patterns, one of these being a circulation between the Rocky Mountains and the plains, with a neutral level (level of reversal of horizontal circulation) near 2,500 m. These data are however not sufficiently numerous to show details of the circulation systems in the region of our interest.

4. Study on the divergence and convergence over the eastern part of U.S.A.

In order to show whether the circulation system discussed in the foregoing paragraph really exists, it would have been desirable to have had data on the mean-wind vectors at different times for a great number of stations. Such data were not available and it was not possible to compute them within the time available. Generally speaking it is not sufficient to compute simple vectorial means from the obser-

vations available at certain levels, since a preference is then given to special circumstances. The most probable vectorial means at higher levels are to be obtainable by the differential method, going step by step up from the surface to higher levels (compare Bleeker 1936, 1938). With the appropriate data the circulation system could have been recognized by computing the horizontal divergence (and convergence) for different levels, and we should then have been particularly interested in the diurnal change of the horizontal divergence (and convergence). There are, however, simple means to get a quick idea of the diurnal change of the horizontal divergence and convergence. It is apparent that:

$$\frac{\partial}{\partial t}\left(\mathrm{div}_2 v\right) = \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) + \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial t}\right)$$

Now the values of $\partial u/\partial t$ and $\partial v/\partial t$ (or rather $\Delta u/\Delta t$ and $\Delta v/\Delta t$) were computed for 4 periods of 6 hr: 4-10, 10-16, 16-22 and 22-04 CST (Central Standard Time); by using all successive observations at the levels of 4,000, 10,000 and 18,000 ft. Thus a 4 CST observation at a certain level was only combined with the *directly following* 10 CST observation; a 10 CST only with the following 16 CST observation, and so on. In this way mean values of $\Delta u/\Delta t$ and $\Delta v/\Delta t$ were computed for all stations for all chosen levels. Patterns of isolines for $\Delta u/\Delta t$ and $\Delta v/\Delta t$ were drawn and from these patterns the values of $\frac{\partial}{\partial x}\left(\frac{\Delta u}{\Delta t}\right)$ and $\frac{\partial}{\partial y}\left(\frac{\Delta v}{\Delta t}\right)$ could be computed and added. Finally

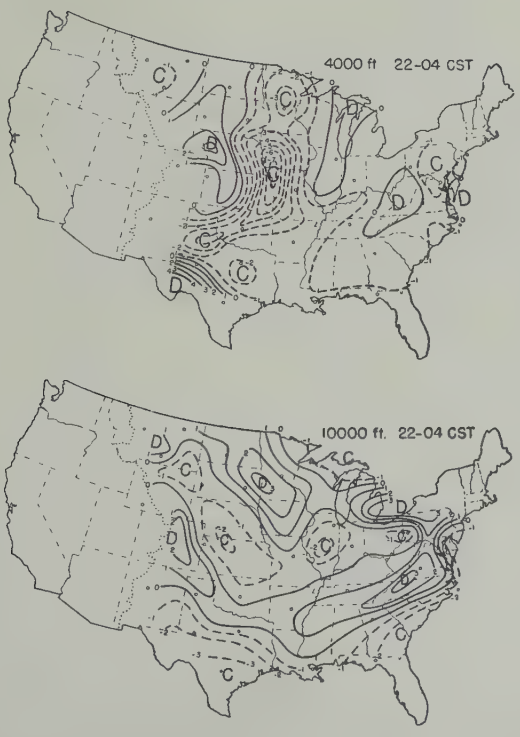

Figure 3. The changes in divergence from 22 - 04 CST

a map for the resultant value of $\frac{\Delta}{\Delta t}\left(\mathrm{div}_2 v\right)$ was drawn. The results of the computation for the months Aug. 1947 and Aug. 1948 are given in Figs. 3, 4, 5 and 6, August being chosen since the nocturnal maximum of thunderstorm activity is pronounced in this part of the year. August in both 1947 and 1948 was relatively hot and dry ; consequently the circulation sought should be pronounced and the more easily detectable since only few pilot-balloon observations ended at low level. Moreover relatively few migratory highs and lows traversed the region of interest during these months. A more extensive sample of summer data might have been desirable but time limitations made it impossible for the authors to extend the investigation. It was, however, possible to check the reliability of the patterns in different ways. In the first place, the two months computed separately showed nearly the same patterns; in the second place it is clear that the sum of the changes for each of the four periods should approach the daily contribution to the seasonal change during the period (which can be assumed negligibly small) for both the component winds and the resulting divergence. In general this condition was fulfilled, with the exception of the higher layer for a number of stations ; data however were rejected or questioned in those cases where the apparent error in the sum, over the four quarter-day periods, of the values $\Delta u/\Delta t$ and $\Delta v/\Delta t$ for that level and station approached or became greater than the smallest mean value of $\Delta u/\Delta t$ or $\Delta v/\Delta t$ in that group. This is a rather rigorous limitation since the values of $\Delta u/\Delta t$ or $\Delta v/\Delta t$ may be really

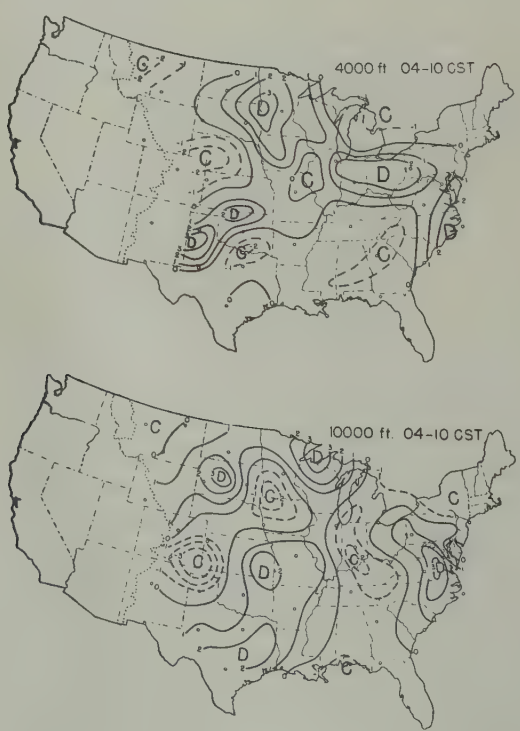

Figure 4. The changes in divergence from 04 - 10 CST

ero. Data from 37 stations, whose locations are shown by circles in Figs. 3-6, were ısed. The figures give the resulting six-hour changes in the divergence field, he units of divergence/time being 10^{-2} hr^{-1} per 6 hr. Letters D and C give the entres of highest and lowest values of change in the divergence.

It appears that there exists a third check on the reliability of the material. One can expect *a priori* that those areas which show an increase of divergence D) in the lower levels should show a decrease of divergence (increase of convergence) C) in the higher levels and *vice versa*. This is indeed the case, and it is particularly lear when we compare the levels of 4,000 and 18,000 ft for the periods of 10-16

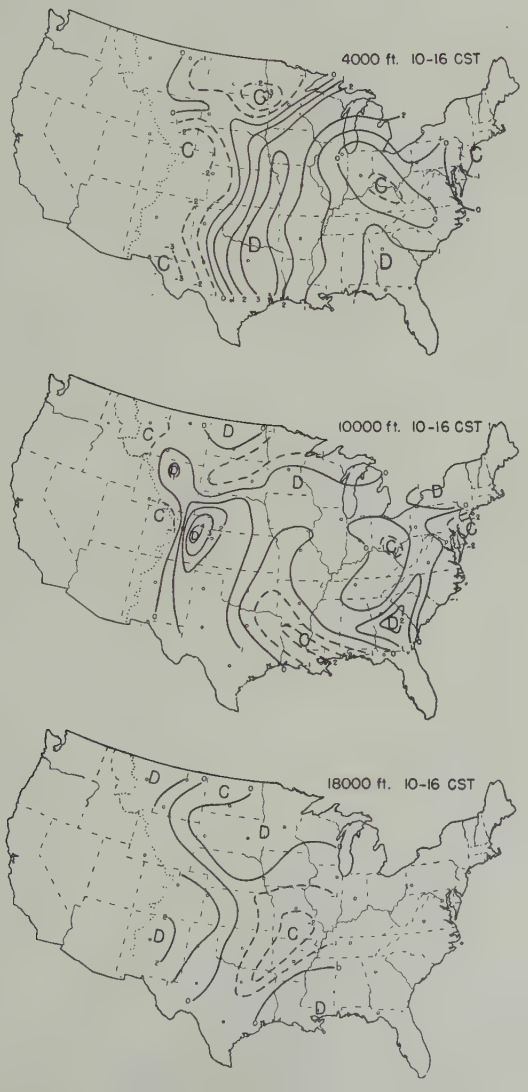

Figure 5. The changes in divergence from 10 - 16 CST

and 16-22 CST within which sufficient data for the higher levels were available. Comparing 4,000 ft and 18,000 ft at 10-16 CST we see that the area over North Dakota and Minnesota which shows decrease in divergence (C) at 4,000 ft shows an increase of divergence D at 18,000 ft ; the elongated strip with increase of divergence (D) at 4,000 ft stretching from Texas, via Oklahoma and Kansas towards Wisconsin is topped by an area of decreasing divergence (C) at 18,000 ft. Comparing 4,000 ft and 18,000 ft of 16-22 CST we again see an excellent anti-parallelism of the D and C areas, with the exception of the area over Wyoming, which however lies at the outer limit of the map, where the patterns are less reliable. Also

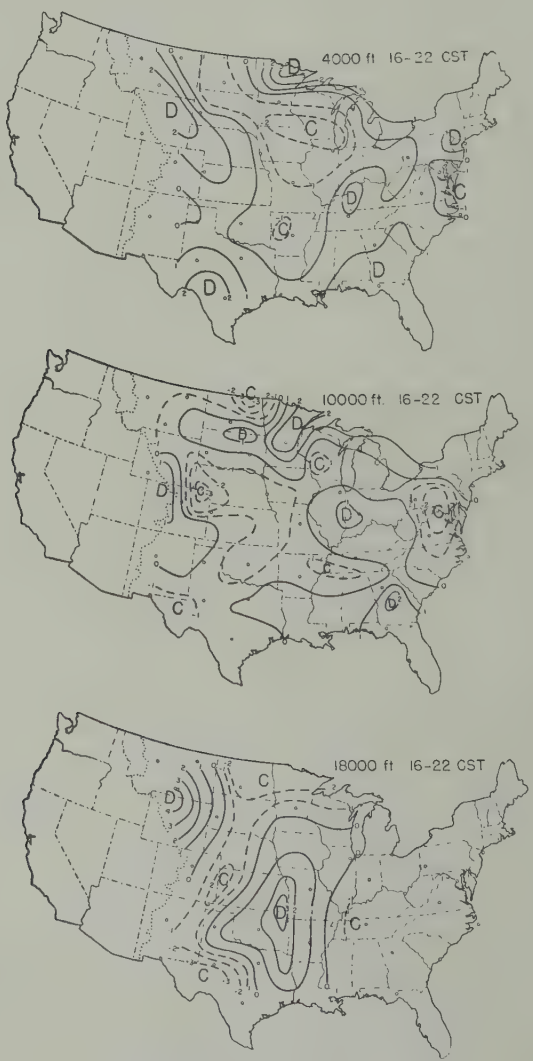

Figure 6. The changes in divergence from 16 - 22 CST

y comparing the 4,000 ft and 10,000 ft patterns for all 4 periods we see that, nerally speaking, areas of increase in divergence are topped by areas of crease in convergence and *vice versa*, though the relation is less well fined, showing that in some areas the level of reversal (level of non-divergence) higher than 10,000 ft.

There is no doubt that the circulation system proposed in Section 3 exists. onsider first the situation of Fig. 6 in which the changes of the divergence from 5 to 22 CST are given. The 4,000 ft level shows a decrease of divergence (increase of convergence) over the plain area, whereas increased divergence occurs ver the whole Rocky Mountains range from over Montana to the Mexican border ; ver the Apalachian area increasing divergence is also present though less clear, ossibly because of the smaller scale of the mountains and the fact that the system f parallel ridges and valleys is more prone to develop numerous local circulations ather than wide-scale effects.

Fig. 3, giving the change in divergence for the principal night period (22 to 4 CST), shows an enormous decrease of divergence (increase of convergence) ver the plain states ; there is still an increasing divergence over the mountain reas west, south and east. The strong decrease of the divergence (increase of onvergence) in the lower levels over the plains means that during the night a real onvergence must be present, which must be coupled to an ascending motion having ts maximum strength in the early morning hours.

From 04 to 10 CST there is at 4,000 ft an increase of divergence over the greater part of the area where previously a decrease was present ; the increase n divergence over the same regions from 10 to 16 CST is conspicuous. At the ame level and during the same periods there is a decrease (increase of convergence) ver the surrounding mountain areas. The increase of divergence in the lower evels over the plains means that during day-time a real divergence must be present, which must be coupled to a descending motion having its maximum strength in the ate afternoon.

5. Comparison between thunderstorm occurrence per quarter-day and the patterns or changes in divergence

It is of interest to compare now the patterns of thunderstorm occurrence in Fig. 1 with the patterns of Figs. 3-6. The reader will note that the data of Fig. 1 on the one hand and those of Figs. 3-6 on the other required the use of different time intervals such that the periods overlap from 2-4 hr. This overlap is partly responsible for the fact that comparison of Fig. 1 and Figs. 3-6 indicates that a significant proportion of the nocturnal thunderstorms occur before the development of maximum divergence. On the other hand thunderstorm activity may be expected to start long before maximum convergence is present, which explains the great area of frequency 30-50 per cent. from 18 to 24 hr Local Time. Whereas most of the thunderstorm activity may occur in the period centred near midnight, it is also reasonable to suppose that the greatest change in convergence in the period 2200-0400 hr CST occurs in the early part, with the greatest temperature variation. In comparing Fig. 1 (18-24 and 00-06 Local Time) with Figs. 6 and 3 we see that they even show

concurrent details. For instance the larger maximum of thunderstorm activity in the period 00-06 Local Time is found in exactly the same place as the maximum increase of convergence in 22-04 CST (4,000 ft). An area of little thunderstorm activity for the 18-24 period over Indiana shows increase of divergence in the 16-22 CST period.

We have not so far mentioned the occurrence of night-time thunderstorms east of the Apalachians. It seems that an analogous circulation system develops in the plain between these mountains and the Atlantic Ocean. The area of maximum thunderstorm activity near the Atlantic Coast (18-24 Local Time) coincides with regions of increasing convergence for the periods 16-22 and 22-04 CST.

In comparing the thunderstorm patterns with the maps of the increase and decrease of divergence one should keep in mind that the thunderstorm patterns are also highly governed by the water-vapour content of the air. The most favourable synoptic situation for the occurrence of night-time thunderstorms of non-frontal character shows a Bermuda High extending to the Atlantic Coast with moist Tropical Gulf air from the south entering the country. It is furthermore not surprising that the frontal thunderstorms also show a night-time maximum (*cf.* Andre 1949). On the whole the factors governing the vertical circulation systems in homogeneous air masses will also be present when fronts cross our areas of interest. And one can expect that non-convective rains also will show the influence of the circulation system on their diurnal variation.

6. Distribution of night-time precipitation in other parts of the world

We have tried to find material from other parts of the world with analogous orographical situations in order to see whether our ideas have a world-wide application but the information required to obtain the diurnal variation of the precipitation as well as the divergence field (or its changes) is not generally available and it has not been possible to add to our American patterns. We were however glad to receive from the Argentine Meteorological Service a map of the occurrence of night-time thunderstorms, drawn by Dr. Wölcken. This map is not based on statistics but on synoptic experience*. Since it fits so completely our ideas, this map is reproduced in Fig. 7. The reader will see that night-time thunderstorms occur in an area at some distance from the Andes. The distance between the watershed of the Andes and the area of maximum night-time thunderstorm activity is even of the same order of magnitude as the distance between the watershed of the Rockies and the centre of night-time thunderstorm activity in U.S.A.

Acknowledgments

Thanks are due to the Director of the Hydrological Section of the U.S. Weather Bureau for permission to reproduce Fig. 1 and to the Director of the Argentine Meteorological Service for Fig. 7. The authors also wish to thank Mr. C. Newton of the University of Chicago for his assistance in discussing the problem and for the drawing of some figures.

* The only data available to the authors on the diurnal variation of rainfall in South America are given by Morandi 1943.

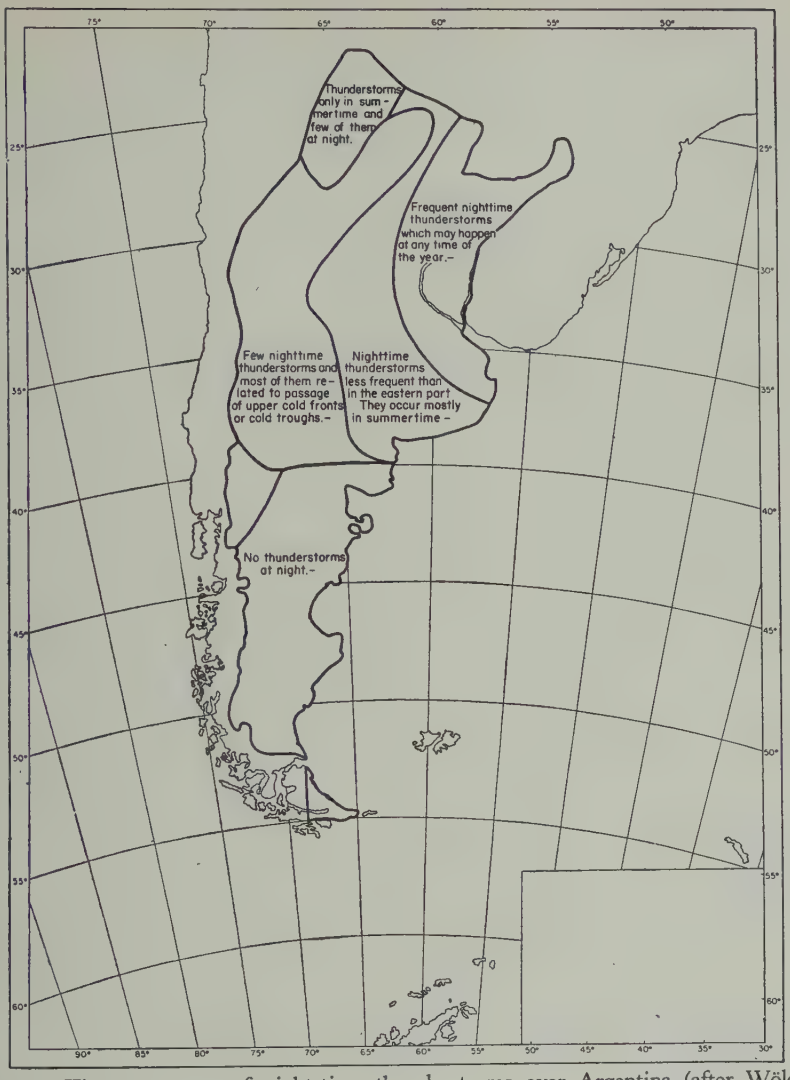
Thunderstorms only in summertime and few of them at night.
Frequent nighttime thunderstorms which may happen at any time of the year.–
Few nighttime thunderstorms and most of them related to passage of upper cold fronts or cold troughs.–
Nighttime thunderstorms less frequent than in the eastern part. They occur mostly in summertime –
No thunderstorms at night.–

Figure 7. The occurrence of night-time thunderstorms over Argentina (after Wölcken).

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The upper-level flow-structure near typhoons

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SUMMARY

From flow charts drawn at the Royal Observatory, Hong Kong, it appears that typhoons commonly develop at airflow discontinuities with Easterlies to the north and Westerlies to the south. Others probably develop within a single airflow, notably N. Pacific Trades, and perhaps N. Indian Westerlies. Typical streamlines in the neighbourhood of typhoons in different localities are presented, and the development and relations of the associated flows are discussed.

1. GENERAL

This paper results from a study of daily flow charts drawn at the Royal Observatory, Hong Kong, during the period September 1947 to November 1949 for the levels 2,000, 5,000 and 10,000 ft and describes the general features common to the typhoons and tropical storms of that period. Observations are rarely, if ever, as copious as the hungry meteorologist desires and consequently only a broad overall picture can be presented. The difficulties of both typhoon research and typhoon forecasting revolve about a shortage of information and for the immediate future no improvement is foreseen. Perhaps the most hopeful approach from the forecasting point of view is the radar technique of Gherzi (1950).

By international agreement three stages of tropical lows are recognized. The tropical depression is the early stage from which intensification may lead through the tropical storm, defined merely by a range of wind velocity, to the full typhoon. Many tropical depressions appearing on the surface charts do not develop to finality, and many circulations are so ineffective and feeble that they hardly merit the title of tropical depressions. Any study of typhoons therefore must account not only for the development of the initial vortex but must explain why it is that some intensify and some do not.

There have been many theoretical attempts to assess the conditions which apply to a fully-developed typhoon that it may maintain its energy. The two most important factors would appear to be instability of the associated air masses, so that latent heat of condensation may be realised, and some form of high pressure in the upper levels so that escape of ascending air may be effected. With regard to instability it must be pointed out that although N. Pacific Trades are involved in the formation of typhoons they do not normally show an inversion towards their periphery. No observational evidence is available in Hong Kong of the very-high-level pressure-conditions associated with typhoons but Riehl (1948) has described observations made in the Pacific. Besides these two conditions others have been suggested and may very well apply and, whatever the truth, it is obvious that the initial vortex cannot develop to typhoon intensity if one or some of the factors are absent.

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Whereas in high latitudes weather emphasis is at density discontinuities (frontal urfaces), equatorwards the emphasis changes to the dominance of convergence, trigger which releases instability. So with the low-pressure systems of the two egions, extra-tropical depressions emphasize density considerations, and tropical epressions, convergence and instability.

2. Tracks and seasons

These are well known and have been most recently summarised by Starbuck 1947). Briefly it may be said that most tropical depressions develop in the area etween the Philippines and longitude 160°E., moving from there in an approximate WNW. direction. Some recurve towards the NE. and others continue their WNW. course. A minority of tropical depressions form in the China Sea west f the Philippines ; some of these recurve, the majority do not.

Tropical storms have been known within the region in all months of the year. February has a minimum with only two recorded tracks since 1884. All other months ave seen full typhoons, the frequency rising to a maximum in August and September.

It is convenient to classify typhoons as :

(1) Typhoons east of the Philippines, where most develop and occur.
(2) Typhoons in the China Sea west of the Philippines, where some develop and many traverse.
(3) Recurving typhoons.

3. The development of the initial vortex

It is undoubted that typhoons develop gradually. In seeking their cause of formation therefore, it is justifiable to look towards those places where the development of small vortices is most likely since obviously a typhoon must show itself initially as a small circulation within the streamline pattern. A probable place therefore is the boundary zone between two currents of air moving in opposite directions. There are two main examples of this within the region, both occurring chiefly between May and October.

The first is the *Intertropical Convergence Zone* (hereafter abbreviated to ITCZ) of summer, a system which develops over the Pacific when Easterly Trades are bordered to the south by the westerly airflow on the northern side of an equatorial clockwise eddy. The second example is the equatorial - N. Pacific Trades discontinuity which is the feature separating two almost zonal currents, the N. Pacific Trades and, to their south, equatorial Westerlies. In August and September this is an almost permanent feature extending across the Pacific and China Seas within the region. During the winter season N. Pacific Trades are zonal across the Pacific, the Philippines and Indo-China and although to their south are equatorial Westerlies, rarely does the separation boundary extend beyond 5°N. Rare also are the occasions of equatorial clockwise eddies in winter though they have been noted north of New Guinea. Thus the occurrence of near-parallel flows of opposite directions fits well with the observed distribution of typhoon origins. It is notable that in every case Easterlies lie to the north and Westerlies to the south.

But it is claimed that vortices may develop within a homogeneous airflow. Riehl (1948) puts forward very strong evidence that this is indeed the case so far as the formation of typhoons is concerned. His study of large numbers of actual observations between the surface and 10,000 ft leads to the conclusion that within the N. Pacific Trades, behind easterly waves, part of the northward-moving air may be deflected to the east, so coming into contact with another part of the westward-moving Trades at a shear line lying approximately from SW. to NE. With deflection of air away from the original wave, vortices may appear along its axis and a typhoon is born, provided conditions in the upper air, which are also described, are suitable. No reason for the shear is offered by Riehl who presents it merely as a fact of observation. It is pertinent to wonder however whether the incipient typhoon really develops on the axis of the original wave, or at the newly-formed discontinuity of opposing air currents which has the same characteristic as noted before of Easterlies to the north and Westerlies to the south. In either case the important feature is the development of the shear in the homogeneous current, and the fact that the vortex is initiated within a single air mass. The density of information available to Riehl was greater than any before or since. It would thus be imprudent to question his findings and it is necessary to accept that a vortex may develop within a homogeneous air mass without the juxtaposition of airflows of different origin.

Royal Observatory flow charts have been studied in an attempt to determine the conditions favourable for typhoon development. Far less information is to hand than Riehl used and only the years 1948 and 1949 could be studied. Knowledge of the existence of typhoons is obtained, not only from the local synoptic and airflow charts, but from warnings broadcast by Manila and Guam based on ship and transport-aircraft reports, and particularly on typhoon-reconnaisance flights from these stations. It is unlikely that any typhoons nowadays go undetected but it certainly cannot be said that all typhoons are followed from their first stages. Generally the forecaster has on his chart a large area of low pressure in which he suspects a typhoon may be forming. He has little or no idea of the exact position of the centre of the disturbance until winds have reached a velocity higher than normal and happen to be reported. Charts two days ahead of the day on which each storm was first warned were scrutinised. This allowed an arbitrary two days for the storm to be initiated, *i.e.* for the vortex to develop and a noticeable circulation to be formed. If the first position for each storm warning falls on a flow discontinuity then there is good empirical evidence that typhoons do actually develop there.

For only 19 storms out of the 44 which were recorded in the two years was there sufficient information and not all of these reached typhoon proportions. Charts for 10,000 ft were used, since they have the most information. The original information was checked before re-plotting and new streamlines were drawn. It was not until all charts had been redrawn that typhoon positions two days later were inserted, in order that there should be no temptation for the charts to be drawn to fit the theory. It was assumed that the storm had not formed at the flow-discontinuity if its first plotted position was more than one degree from the position of the discontinuity, though it is considered that this is an extremely harsh criterion.

The results of this investigation were :—

Formation at equatorial - N. Pacific Trades discontinuity.	
(1) over Pacific	6
(2) over China Sea	2
Formation at ITCZ	
(1) over Pacific	2
(2) over China Sea	1
Formation within N. Pacific Trades	7
Formation within N. Indian Westerlies	1

Eleven storms developed within one degree of the drawn position of an airflow-discontinuity, which was always oriented approximately east to west and had Easterlies on the north side and Westerlies on the south side. Fig. 1 is an example of such a chart. The discontinuity separates N. Pacific Trades from two Westerly flows, *viz.* equatorial or N. Indian Westerlies reaching across the northern Philippines, and S. Pacific Trades brought across the equator by an equatorial clockwise eddy. Thus the chart illustrates each of the two flows which may, and usually do separately, form the discontinuity with the Trades. That the position where the typhoon developed on 12 Sept. might be considered as a " triple point " is thought to be coincidental.

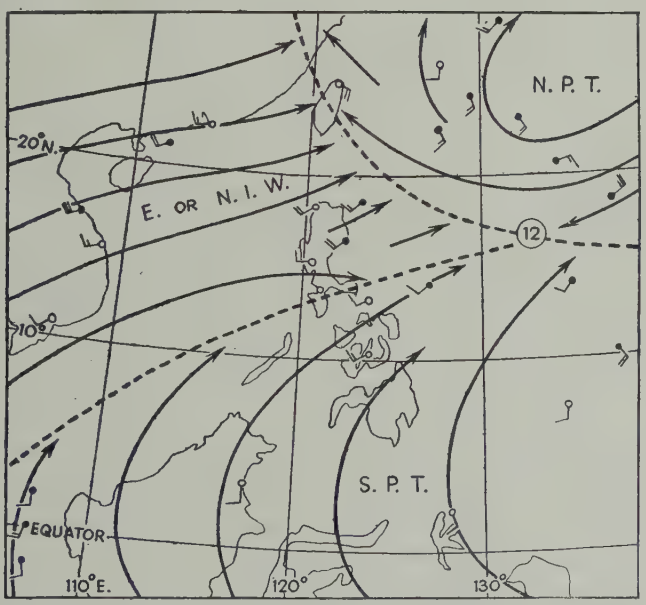

Figure 1. 10 September 1948 at 10,000 ft. Typhoon development on 12 Sept. at an airflow discontinuity. Plain circles, pilot balloons. Black circles, aircraft reports. One full flèche represents 10 kt.

N.P.T. — N. Pacific Trades.
E. or N.I.W. — Equatorial or N. Indian Westerlies.
S.P.T. — S. Pacific Trades.

A further example is given in Fig. 2. The storm developed at the equatorial N. Pacific Trades discontinuity, Trades to the north and equatorial Westerlies to the south. Although the storm which formed on 20 Aug. did not reach typhoon proportions this situation is particularly dangerous to Hong Kong. In September 1949 with an almost identical situation a storm developed off the NW. Philippines. It intensified and accelerated rapidly and passed 50 mi to the south of the Colony, fortunately causing only minor damage, despite winds of 70 — 80 kt outside the harbour. Fig. 2 should be compared with Fig. 3 which is quoted by both Jeffries and Heywood (1938) and Starbuck (1947). The former state, " Typhoons originate in elongated areas of slight barometric depression which lie with their major axes

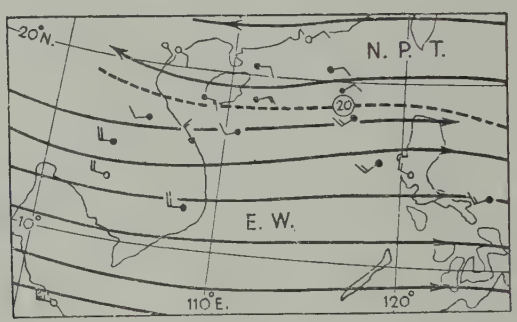

Figure 2. 18 Aug. 1948 at 10,000 ft. Tropical storm development on 20 Aug. at an airflow discontinuity ; the equatorial N. Pacific Trades discontinuity.
E.W. — Equatorial Westerlies.

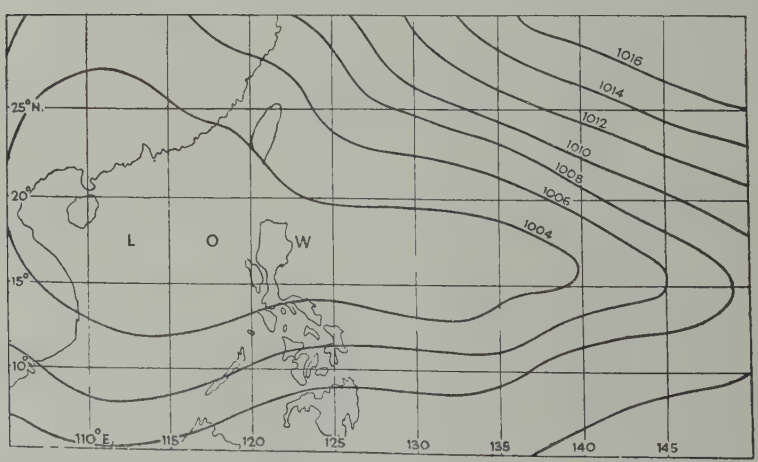

Figure 3. 0600, 17 August 1937. A typical midsummer pressure-pattern which is frequently th birthplace of typhoons. In the period 17 August to 3 September 1937, four separate circulation centres were located in this trough including the very severe Hong Kong typhoon of 2 Septemb when a gust wind-velocity of about 170 mi hr^{-1} occurred. (from Starbuck 1947).

E. and W. or ENE. and WSW. across the China Sea, the Philippines and the Pacific eastwards to the Caroline Islands." An airflow pattern such as that of Fig. 2 would be represented by an isobaric distribution similar to that of Fig. 3, though the former chart is of 10,000 ft and the latter of the surface. The trough is the pressure representation of two airflows of opposite direction.

No development occurred at the ITCZ in the China Sea when it had its normal structure of N. Indian Westerlies on the north side and the Westerlies of recurving S. Pacific Trades on the south side. One storm however did form at this feature when it separated Trades to the north from recurving S. Pacific Trades to the south. This was the only time of the two years that an ITCZ of the China Sea had opposing currents on either side.

One depression, not included in the above summary, developed at a shallow ITCZ over the Pacific near Mindanao. It remained weak throughout its life and eventually filled up over the China Sea. This storm formed at the junction of a clockwise eddy current and N. Pacific Trades at the 2,000 ft level, but at 10,000 ft there persisted an Easterly current only. The hint is therefore that unless opposing currents have a depth of at least 10,000 ft a small circulation appearing near the ground will not intensify. This suggestion is supported by the observation that equatorial clockwise eddies are frequent at 2,000 ft but rare at 10,000 ft. Similarly the frequency of closed isobars on the surface chart is much greater than the occurrence of typhoons.

It must be made quite clear that there is no suggestion here that typhoons develop at fronts in the same way as the depressions of high latitudes. Though flow discontinuity is observed, density discontinuity is uncertain and awaits more complete upper-air sounding. Furthermore it is too early yet to say that with all cases of airflow-discontinuity to 10,000 ft a typhoon will necessarily develop. Presumably there are other critical factors. More information is awaited.

Seven storms formed more than one degree from a line of flow discontinuity but in five cases there was a flow discontinuity within three degrees. It is felt therefore that the evidence does not much uphold Riehl's observations, since in these five cases there is the possibility that the storm had developed on the discontinuity but had moved as a feeble undetected system within the two-day period. It is more than likely therefore that the proportion of typhoon developments given as within N. Pacific Trades is far too high. However in Fig. 4 one of the remaining two charts is presented as drawn, and although information is scanty east of the Philippines, and the location of the convergence zones is by no means certain, at least it is most unlikely that there is a flow discontinuity where the typhoon developed. Perhaps the circulation at 10,000 ft developed in the position where it appears to be forming on this chart. This would mean a movement of some 660 mi in about 48 hr to bring it to the position of the surface low on the 24th. Not improbable perhaps.

Further evidence that storms may develop within one airflow only is offered by the one storm which appeared to form over the China Sea within a weak flow of N. Indian Westerlies (26 July 1949). Remarkably enough this is the only storm whose initial movement had an easterly component of motion, all others moving initially towards the west to a greater or less degree.

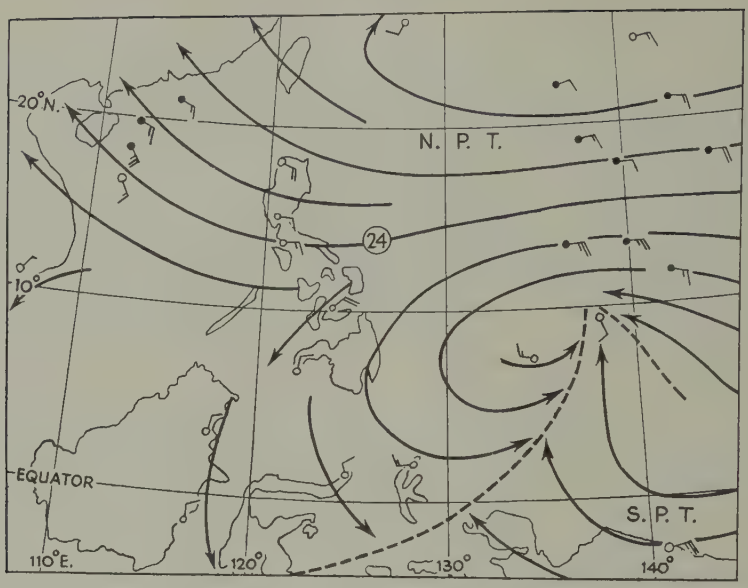

Figure 4. 22 Sept. 1948 at 10,000 ft. Typhoon development on 24 Sept. within N. Pacific Trades in the position shown.

4. The streamline structure of typhoons

A striking feature of some typhoons is the comparative smallness of the hurricane-wind area and in fact, very often, the smallness of the whole system. It is not uncommon to find, when a typhoon circulation is on the chart, that the strong winds and bad weather are localised near the storm centre and the airflows approaching from the wider area have little more than their normal velocity. This however is not always the case.

Aircraft winds near the centre of typhoons cannot be considered accurate and consequently the exact structure of the flow near the centre is uncertain. A uniform vortex is generally assumed but it is unlikely that this is true. If the vortex were uniform the eye should be circular. This is not necessarily the case. The reply of a United States typhoon-reconnaisance pilot who had flown in many centres, when questioned on this point was, "it may have any shape or size, dumb-bell, oval, circular, or shapeless." Humphrey and Fite (1945) state that " it is usually egg-shaped, elongated in the direction of motion, but approaches the circular as the storm develops." However the typhoon which crossed Hong Kong on 23 November 1939 had a calm area, oval in shape, four miles wide and nine miles long, the shorter axis being parallel with the direction of motion of the typhoon (Heywood 1940). Towards the periphery a uniform circulation is certainly not the case as may be seen from any of the figures.

Fig. 5 is an example of a type of flow structure common east of the Philippines. The development of the structure can be readily appreciated in the case of a storm forming on an ITCZ. If the storm forms on the equatorial - N. Pacific Trades discontinuity the Pacific trans-equatorial flow may not develop at all. In the case of storms appearing to form within the Trades it is not long before this southerly flow (or equatorial Westerlies) has been initiated and has reached to the centre of the

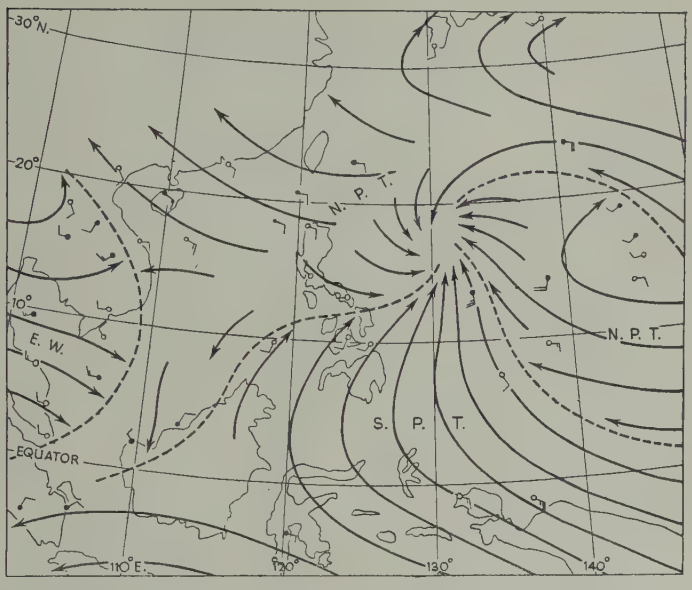

Figure 5. 21 July 1949 at 10,000 ft. Typhoon with flows of N. and S. Pacific Trades. Note the divergence area to the west.

circulation. Since some storms appear to develop within the Trades it is evident that the presence of the trans-equatorial current is not an essential requirement for storm development. It is a secondary development and it is observed as such. Figs. 2 and 7 show that it may not appear at all.

The convergence lines should not be regarded as fronts since density discontinuity cannot be proved. Nor, particularly in the case of the easternmost feature, can they always be exactly located since the direction of the flows on either side is often very much the same, and convergence is comparatively slight over a broad area rather than strong over a narrow line. Towards the centre, of course, they are obscured in the general spiralling convergence.

As a typhoon with a southerly flow moves west the southerly current appears to move bodily with it, air being attracted from across more and more western sectors of the equator and ceasing to flow from the more eastern, where the normal flows, though probably intensified, re-establish themselves. The convergence zones do not appear to change their positions relative to the typhoon. Ultimately, towards the west, equatorial Westerlies feed into the circulation and a secondary, though often indeterminate, convergence zone comes into being between the two flows (Fig. 6). With further westward movement of the typhoon beyond the Philippines the southerly trans-equatorial flow generally dies out altogether.

Fig. 7 is a flow chart of a typhoon which was pursuing a course from N. Luzon to N. Hainan. In this case the southern sector is composed entirely of equatorial or N. Indian Westerlies, there being definitely no trans-equatorial flow. This type of structure is quite common east of the Philippines as well as in the China Sea, occurring in almost all cases of typhoons developing at the equatorial - N. Pacific Trades discontinuity. A perhaps surprising feature, and one that has been noted

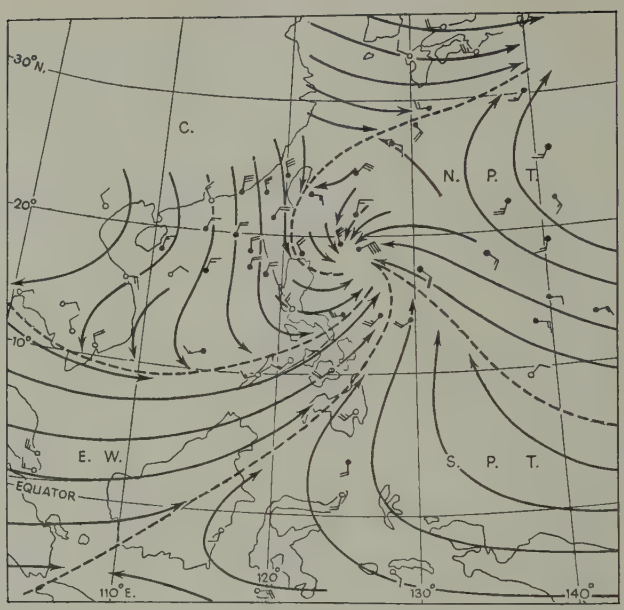

Figure 6. 26 Oct. 1949 at 10,000 ft. A secondary convergence zone is introduced between S. Pacific Trades and equatorial Westerlies feeding into the southern sector of a typhoon into which continental air (c) is also fed.

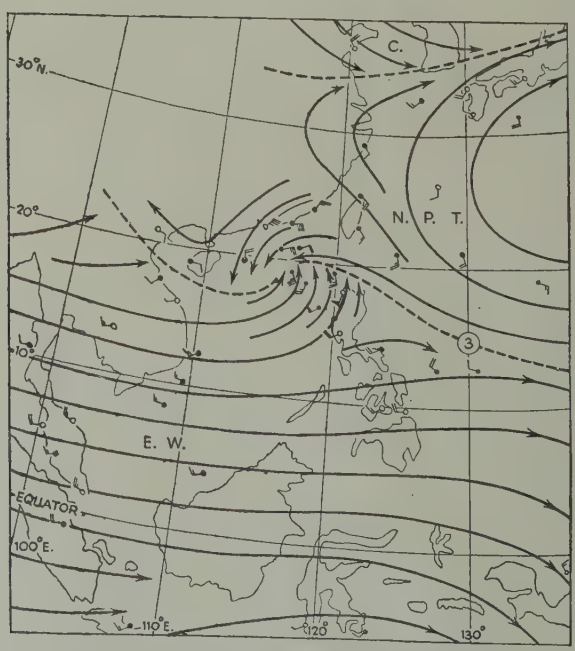

Figure 7. 2 Sept. 1948 at 10,000 ft. Typhoon with flows of equatorial or N. Indian Westerlies and N. Pacific Trades only. Note the divergence area to the west of the centre. Another typhoon developed on 3 Sept. in the position shown.

several times, is the turning away of equatorial Westerlies to the SE. of the typhoon. It might better be said that the equatorial Westerlies are maintaining their general flow and only a small part is converging into the typhoon. It is notable that at the convergence zone between Westerlies and Trades (the equatorial-N. Pacific Trades discontinuity) east of the Philippines a new typhoon developed on 3 September.

It will be observed that the southern sector of the typhoon of Fig. 7 is wide, whereas that of the typhoon shown in Fig. 8 (not the same typhoon as that in Fig. 7) is narrow. Southern sectors remain wide whilst typhoons are in low latitudes but always the sector shrinks if a storm moves northwards, until finally, near the latitude of Japan, a single convergence zone exists between southerly or south-easterly N. Pacific Trades, and Trades which have fed to the west of the storm. This convergence, by reason of the different recent history of the airflows, may be slightly frontal. But, in these higher latitudes, there is another factor to be considered, namely the approach and incorporation of new flows. These flows are the high latitude Westerlies or north Westerlies. The typhoon of Fig. 8 is approaching this airflow and the process of its incorporation to render the final system strongly frontal and extra-tropical in characteristics can readily be envisaged. In Fig. 6 such an incorporation has already taken place at low latitudes.

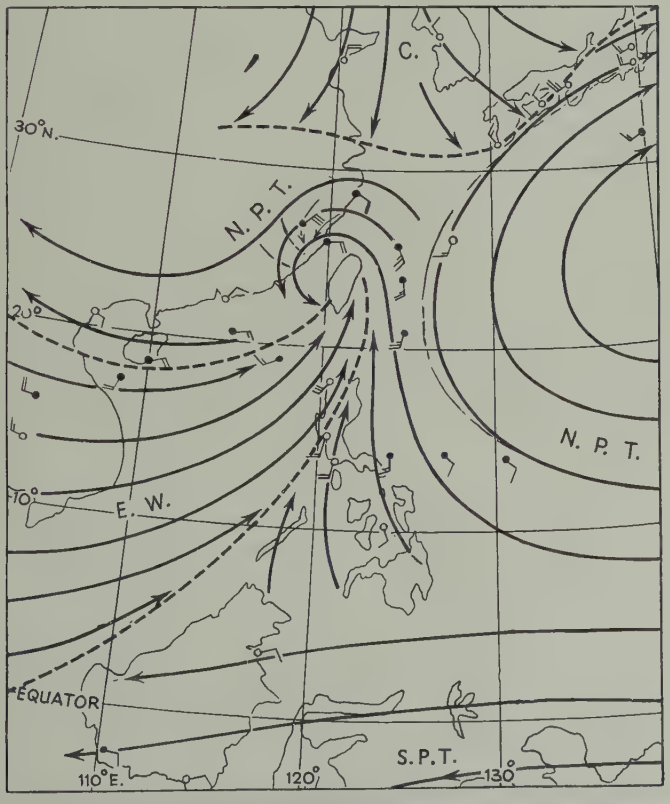

Figure 8. 6 Sept. 1948 at 10,000 ft. Typhoon moving northwards with narrowing sector of equatorial Westerlies. Note the divergence area just to the west of the typhoon.

A striking feature of all the typhoons studied is the appearance and behaviour of the N. Pacific Trades. This is a massive flow and only a small part, comparatively speaking, appears to enter the storm, the majority following the track determined by the state of development of the sub-tropical N. Pacific anticyclone of the moment. In the E. and NE. sectors of the typhoon there is widespread convergence within the Trades. In the W. or NW. sectors however conditions are very different. That part of the Trades which turns into the typhoon is abruptly divergent from its original direction to a northerly or north-westerly flow, and accordingly there results, W. or NW. of the typhoon centre, an area of extremely good weather with light and variable winds. These areas can be picked out easily in all the Figs. 5 — 8. Heywood (1933) has listed pilot-balloon observations made at Hong Kong in the neighbourhood of typhoons which show this feature extremely well. The divergence is so strong that it may sometimes be seen on the surface charts, winds to the W. being NE. and those to the E., N. or NW. Pre-typhoon divergence is so typical that the general public in Hong Kong associates bright, dry weather with perhaps a few fair-weather cumulus clouds and light north-easterly winds as a harbinger of typhoons which nearly always approach the Colony from the E. or SE. Jeffries and Heywood (1938) state, " In Hong Kong a period of fine weather is almost invariably associated with a distant typhoon to the eastward or south-eastward." Eastwards from the divergence-area, convergence is rapid, winds increase and weather deteriorates, sometimes abruptly.

It has been frequently stated that there is a fairly constant inflow of wind towards the centre of typhoons of approximately 30° and this is one method adopted to locate the direction of the storm centre. Whilst the precept may be true of the convergence area it obviously does not hold within the divergence area and care must therefore be taken that winds are actually within the inflow area before their direction is used to estimate the direction of the centre of the typhoon.

Acknowledgments

I am indebted to Mr. G. S. P. Heywood, Director, Royal Observatory, Hong Kong, for permission to publish this paper, and for very helpful criticism and advice.

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Observations with a directional raingauge

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(Manuscript received 1 November 1950)

Summary

An octagonal raingauge, which was set up to study the way in which rain impinged on vertical surfaces, is described. Observations made over a period of 12 years are analysed, showing that, in the London area, walls with moderate exposure to wind and facing approximately SW. receive most rain, equivalent to about 1/4 of the normal rainfall, and walls facing N. receive least, about 1/16 the normal rainfall. An equation connecting the mean incident angle of the rain on the ground with the wind speed is derived, and an attempt is made to obtain an indication of raindrop diameter.

1. Introduction

In the course of work on the properties of building materials it became necessary to determine how much rain strikes the surface of vertical walls facing in different directions. In view of the erratic nature of the air currents round buildings the most precise method would be to set collecting apparatus on the faces of actual buildings. There would then be the disadvantage that the results would only be applicable to the particular building studied and would not necessarily be of general value. The alternative method has been adopted therefore of setting up a special raingauge, with eight vertical collecting apertures facing the cardinal points, in addition to the normal horizontal aperture. The objective is to obtain data which, while not necessarily directly applicable to the walls of a building, will at least give some indication of the amounts received in practice.

After observations had been started with this gauge our attention was drawn to the work of Chrimes (1872), who had set up a gauge with four vertical trumpet-shaped collectors at Rotherham, Yorkshire in 1866, and to some even earlier experiments by Professor Phillips at York. The measurements made with Chrimes's gauge were used to compute estimates of mean incident angles of rainfall for monthly periods. The observations continued for a number of years after 1872, but were discontinued when the gauge became unserviceable.

Fourcade (1942) proposes a gauge with four vertical or inclined apertures for determining the inclination of falling rain, and illustrates such a gauge which had been set up in 1941 at Jonkershoek in South Africa. A few preliminary observations made with the gauge are quoted.

The gauge described in this note has been set up on the flat roof of a building at Garston, near Watford, Hertfordshire, and the readings obtained during the period 1937 to 1949 inclusive are presented and discussed.

2. Description of the gauge

At the time this investigation was proposed no previous work on the subject was known, and the gauge described in this note was therefore designed by H. E.

Beckett in 1936. It was built in the workshops of the Building Research Station, and was described, together with the observations made in the first year, in a Building Research Station note. (Beckett 1938).

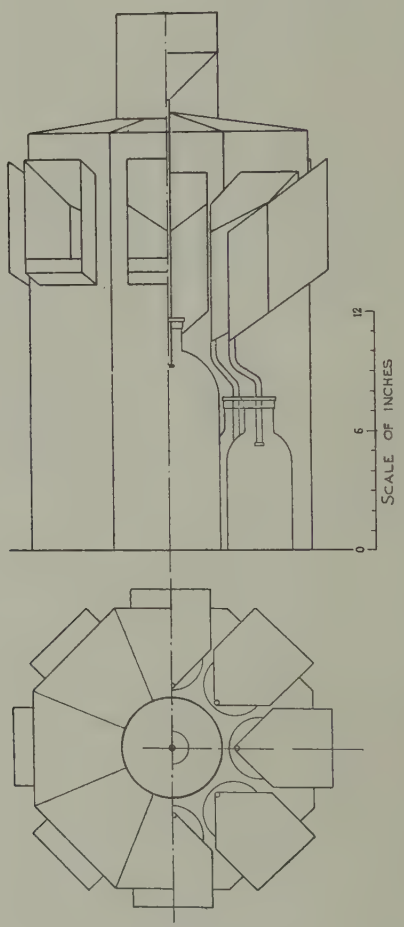

Figure 1. Part-sectioned plan and elevation of directional raingauge

The gauge is made of sheet copper, painted white except within the collecting apertures, and is illustrated in Fig. 1. It is octagonal in plan and has a rectangular collecting aperture measuring 5 in. by 4 in. in each vertical face, designed so as to minimise the risk of loss of rain due to splashing. The lowest part of the vertical apertures is some 15 in. above ground level, so that there should be little risk of any appreciable quantity of rain splashing into them from the ground. On the top of the gauge is a circular horizontal collector, also 20 in^2. The nine collectors drain through copper tubes into bottles within the gauge. When the apparatus is assembled the mouths of the bottles are covered by circular brass caps sliding on the ends of the drainage tubes. Small plates are fixed below the ends of the drainage tubes,

nd are intended to prevent air currents blowing down the tubes and displacing he paraffin which is used to restrict evaporation from the bottles.

Two sets of bottles are provided, together with a metal basket to hold the pare set, one set being marked in red paint, the other in blue, with letters N, NE, *tc.*, corresponding to the directional collectors, to prevent confusion when changing he bottles.

3. Experimental procedure

The gauge is set on the flat roof of a building, 13 ft above ground. The parapet, ft high, is nowhere closer than 17 ft and the exposure is good in all directions.

The amounts of rain collected are normally read at monthly intervals, but the atch is occasionally greater than was allowed for in the design of the instrument, and t is then necessary to change the bottles before the end of the month. The spare set of bottles which is to be put into the gauge is prepared by putting in each of the ight directional bottles 50 ml of water and 5 ml of paraffin, and in the other bottle 00 ml of water and 10 ml of paraffin. Experiments with dummy gauges showed that he paraffin (burning oil) cut down evaporation by over 70 per cent. when deflector lates were used to prevent displacement of the paraffin by air currents. The added vater is necessary to give initially a satisfactory film of paraffin. The bottles are xchanged on the morning of the first day of each month, unless because of excep- ional rainfall it is necessary to change them sooner, so that the totals obtained can e compared with the readings of the standard raingauge set up in the meteorological nclosure some 150 yd to the N. The volume collected in each bottle is measured nd converted into equivalent inches of rainfall.

A cup anemometer with electrical contacts is set up on a mast 20 ft W. of he raingauge, the anemometer head being about 11 ft above roof level, and the un of wind is continuously recorded in the laboratory below. This recorder, in conjunction with the Dines rainfall recorder, enables the mean wind speed during he period of rainfall to be computed. The wind speed so obtained is converted nto wind speed past the raingauge at a height of 2 ft, by means of a calibration curve obtained with a sensitive cup anemometer.

4. Summary of the observations

Fig. 2 shows the mean annual amount of rain, expressed in equivalent inches of rainfall, collected in each of the vertical apertures of the gauge during the period 1937 to 1948. The variation from year to year is expressed as the standard deviation for each of the values, indicated on the figure by vertical lines equal in length to the standard deviation, above and below each point.

In Fig. 3 the mean catches in each of the four seasons of the year are shown, the seasons being defined as :

Winter = December-February ; Spring = March-May ;
Summer = June-August ; Autumn = September-November.

This figure covers the period Dec. 1936 to Feb. 1949, giving twelve complete sets of seasons, the winter 1945-1946 being omitted as the results were incomplete.

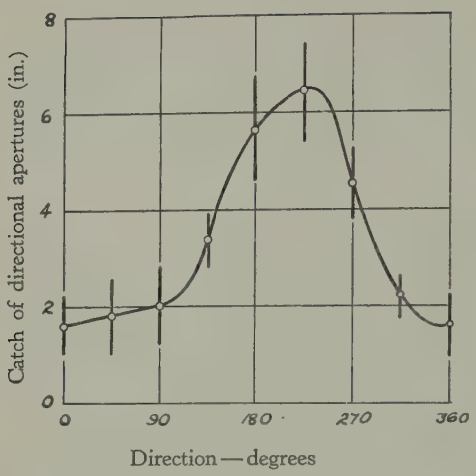

Figure 2. Mean annual catches in vertical apertures 1937-1948.

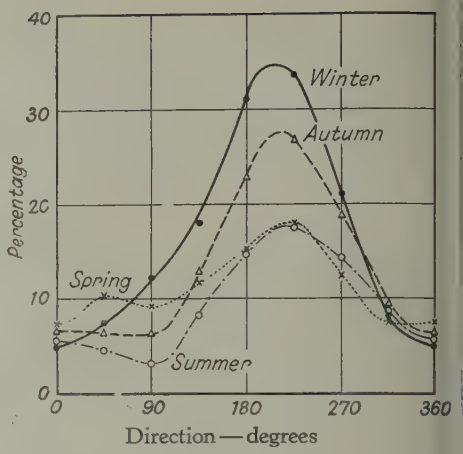

Figure 3. Mean catches in vertical apertures by seasons, expressed as percentage of catch in horizontal aperture.

Fig. 2 shows that an aperture facing approximately 230°, *i.e.* slightly W. of SW., would have received most rain during this period, and one facing 350°, *i.e.* slightly W. of N., would have received least. The ratio of the amounts caught in two such apertures would have been about 6·5 to 1·55, or 4·2 to 1. The equivalent depths of rain caught, 6·5 and 1·55 in. respectively, represent 25·1 per cent. and 6·0 per cent. of the mean annual rainfall during the period. It will be seen that the curve in Fig. 2 is by no means symmetrical about the maximum and minimum points.

Considering now the catches in the vertical apertures during the different seasons, as shown in Fig. 3, it is seen that the distribution as between the apertures varies from season to season. In summer and autumn the east aperture receives least rain, whereas in winter and spring the north receives least. In all seasons the SW. is the wettest side. The ratios between the catches on the wettest and driest sides are : Winter 7·0 to 1, spring 2·4 to 1, summer 5·6 to 1, autumn 4·6 to 1.

From Fig. 3 it is clear that the amounts of rain caught in the vertical apertures do not bear a constant ratio to that caught in the horizontal aperture, *i.e.*, the true rainfall. In summer from 3 per cent. to 18 per cent. of the true rainfall is caught in the individual vertical apertures, and in winter from 5 per cent. to 35 per cent. This seasonal variation is due to variations in the incident angle of the rainfall, which is mainly caused by variations in wind speed. In section 6 below we will consider how the incident angle of the rain may be measured by this gauge.

While we cannot apply the results from this gauge directly and quantitatively to actual buildings, owing to differences between the air flow past this gauge and that past a large structure, it is likely that the results are qualitatively true for large structures. We can say therefore that in places with moderate exposure to wind, the quantity of rain falling during a year on the wettest wall of a building amounts to approximately 1/4 of the rainfall on the ground. The amount falling on the driest side is about 1/4 of that falling on the wettest side. A wall facing approxi-

mately SW. would receive most rain on it, one facing approximately N. would receive least. Walls exposed to stronger winds than those experienced at the site of this gauge would receive a greater proportion of rain upon them.

5. Comparison of directional and standard gauges

As the directional gauge is not ideally exposed on a flat roof the readings of the horizontal aperture were compared with those of the standard gauge in the meteorological enclosure by computing for each month the difference between the readings of the two gauges. Over the whole twelve years the mean difference was 0·10 in. per month, the sign of the difference not being taken into account. On the average the differences were greatest in winter, the mean value for December to February being 0·15 in., that for June to August being 0·05 in. Expressed as percentages of the actual rainfalls, the corresponding values were 7·0 per cent. and 2·8 per cent., so that the difference between the two gauges was proportionately greater in the winter months. It is probable that this is mainly because of the effect of the stronger winds experienced in winter, but it is in part because on occasion the directional gauge apertures became choked with snow, and most of the overall deficit in the catch of this gauge is attributable to this cause.

There is a slight tendency for the readings of the horizontal aperture of the directional gauge to be lower than those of the standard gauge in the winter months, but apart from this the positive and negative differences appear to be scattered in a random manner. This suggests that they are in part at least due to local variations in the rainfall, the two gauges being about 150 yd apart.

The total catch in the horizontal aperture of the directional gauge in the twelve years' observations is 2·27 in. below that in the standard gauge, the difference being 0·7 per cent. of the total. Most of this difference being due to loss of snow on a few occasions of exceptional snowfall and not attributable to the exposure of the gauge, it is concluded that the directional gauge is not exposed to the wind to such an extent as to give an unrepresentative measure of the rainfall.

6. The measurement of the incident angle of rain

During their passage through the air the raindrops follow a path which is in general inclined to the vertical, the inclination depending upon the vertical velocity of the drops and the speed of the wind. Near the ground the vertical velocity of each raindrop will normally be the terminal velocity corresponding to its size.

Consider first the case of a shower of rain with uniform drop-size, in a wind of constant speed, so that the incident angle i of all the drops is the same, and a gauge with two apertures each of area a, one aperture being in a horizontal plane, the other in a vertical plane normal to the direction of the wind. (Fig. 4).

Then the horizontal aperture collects a column of rain of cross-section $a \cos i$, measured normally to the direction of incidence, and the vertical aperture collects a column of cross-section $a \sin i$. The incident angle is then given by :

$$\tan i = \text{catch in vertical aperture/catch in horizontal aperture} = a \sin i / a \cos i \ . \quad (1)$$

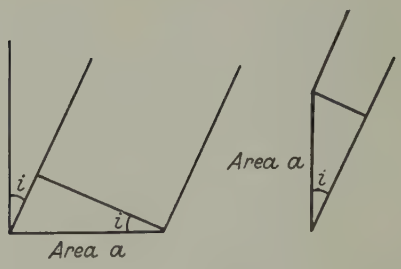

Figure 4. Rainfall catch in horizontal and vertical apertures.

In general however a shower of rain consists of drops of a number of different sizes, and owing to these variations in size, and to variations in wind speed, the drops in any given shower will fall with a number of different incident angles. The true mean angle of incidence for the shower is the mean of all the individual incident angles, but it is not possible to measure this directly.

Let the incident angles of the various sizes of drops be $i_1, i_2 \ldots i_n$, and let the volumes of rain in the column of unit cross-section associated with each angle be $r_1, r_2 \ldots r_n$. The mean incident angle $\bar{i}$ is given by

$$\bar{i} = \Sigma(ri)/\Sigma r \qquad (2)$$

The mean tangent of all the incident angles for the shower, $\overline{\tan i}$, is given by meaning the weighted tangents of the individual angles, so that ;

$$\overline{\tan i} = \Sigma(r \tan i)/\Sigma r \qquad (3)$$

Once again however, these individual quantities cannot be measured directly, but for the directional raingauge we have the following relationships :

Catch in the vertical aperture $= a\Sigma(r \sin i)$ and,

Catch in the horizontal aperture $= a\Sigma(r \cos i)$

Assuming that the ratio of these quantities gives the mean tangent required we have

$$\overline{\tan i} = \Sigma(r \sin i)/\Sigma(r \cos i) \qquad (4)$$

To check whether the angle computed from Eq. (4) was significantly different from that obtained from the other two equations, a comparison was made using raindrop size-distribution data quoted by Laws and Parsons (1943) and raindrop terminal velocities given by Best (1950). A mean rainfall intensity of 0·05 in./hr was assumed, and a wind speed of 5 m/sec. The sine and cosine of the incident angle were taken to be proportional respectively to the wind speed and the raindrop terminal velocity. From Eqs. (2), (3) and (4) the incident angle was computed to be 45·9°, 46·8° and 45·6° respectively. Considering the likely inaccuracies in the measurements it seems clear that it will be sufficient to take the ratios of the catches in the vertical and horizontal apertures as the mean tangent of the angle of incidence.

However, owing to the construction of the gauge, even in a steady wind there may be no vertical aperture normal to the wind direction, while in general the wind direction will vary continuously about the mean value. In a perfectly steady wind, with the present octagonal gauge, the rain will be caught in three of the vertical apertures if the wind direction is normal to one aperture, and in four apertures if the wind comes from any other direction. The area of the vertical apertures, each of area a, projected on to a plane normal to the wind direction varies with wind direction from 2·40 a to 2·61 a, with a mean value of 2·53 a. If we may assume that the raindrops are not deflected by the air currents round the gauge, the total catch in the vertical apertures during a fall of rain with a steady wind will be 2·53 R, if R is the amount that would be caught by a vertical aperture of area a normal to the wind direction.

This argument may be extended to the case of a wind varying in direction by considering separately the rain caught when the wind is blowing from any particular direction. In each case the total catch will be 2·53 times that which would have been caught by a vertical aperture normal to that direction. The total amount caught will therefore be 2·53 times that which would be caught in a vertical aperture which was always normal to the wind direction.

It follows that the mean angle of incidence, according to our definition, is given, in the case of the octagonal gauge, by :

$$\tan \bar{i} = \text{total catch in vertical apertures}/2{\cdot}53 \div \text{catch in horizontal aperture.}$$

This result applies if the catch is that from one shower only, or that from a number of showers with the wind blowing from a different direction during each shower.

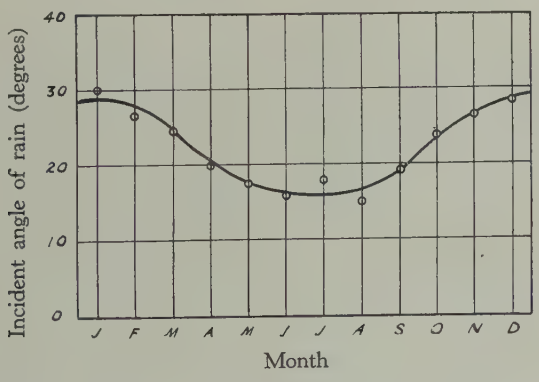

Figure 5. Variation in angle of incidence of rain with time of year.

The 12-year mean incident angles of the rainfall for each of the twelve months have been calculated in this way, and the values are plotted in Fig. 5, a smooth curve being drawn through the points. The curve resembles a sine wave with a maximum in mid-January and minimum in mid-July. The mean value of the incident angle over the year is 22·5°, with a seasonal amplitude of $\pm$ 6·5°. The extreme values that have been measured in an individual month are 7° and 46°.

These values may be compared with those quoted by Chrimes for the seven-year period 1866-1872 at Rotherham, *i.e.* a mean incident angle of 38·3°, with a mean annual range of 37·5° between the maximum in March and the minimum in July. These figures are much greater than those obtained from the Garston gauge, and it seems likely that the Rotherham site was much more exposed to wind.

It would be of considerable use if an estimate could be made of the amount of rain that would strike a given wall in a shower of rain, assuming the wind speed to be known. Owing to the variation from shower to shower of the drop-size distribution, it is not possible with the information at present available to do this for an individual shower of rain, but monthly values of the incident angle of rainfall, together with the mean wind speeds, have been computed for the period Feb. 1948 to Feb. 1950. For these 25 sets of observations a regression equation connecting tan$i$ with v, the wind speed during the rain at the height of the raingauge apertures, has been worked out :

$$\tan i = 0{\cdot}04 + 0{\cdot}140\, v \quad . \quad . \quad . \quad . \quad (5)$$

The residual standard deviation is 0·109, which at a mean incident angle of 22·5 degrees represents about 5·5 degrees. Part of this variation is due to experimental error, but probably the greater part is because the drop-size distribution does in fact change from month to month. The data used in the calculation of Eq. (5) is plotted in Fig. 6, together with the regression line represented by the equation.

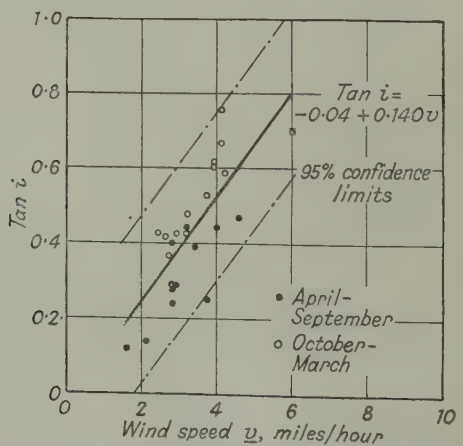

Figure 6. Variation in tangent of incident angle with wind speed.

The broken lines drawn above and below the regression line in Fig. 6 indicate the 95 per cent. confidence limits. That is to say, it may be expected that 19 observations in every 20 will lie between these lines. It will be seen that 24 of the 25 points lie within the lines.

Eq. (4) applies to rain falling on a small object that does not cause appreciable distortion of the air flow. A large structure such as a building will affect the air flow considerably, and the air speed near the walls will usually be less than that in

he free air, but the relationship between the wind speed in the open and that near wall will depend on the orientation of the wall as well as the size of the structure. is probable therefore that the use of Eq. (4) with the wind-speed measured in the pen air, will give an over-estimate of the incident angle of the rain, and therefore f the amount of rain intercepted by a wall.

7. Raindrop size

Using the rainfall incident angles obtained in the way described above and the orresponding wind speeds, it is possible to calculate for each month the mean ertical terminal velocity of the rain, and hence the corresponding raindrop-size. Ve shall term this drop-size the " equivalent drop-size," and it is such that if all he rain that fell consisted of drops of the equivalent drop-size, then the incident ngle would be equal to that actually measured. The values of the equivalent drop-iameter D_e for each of the 25 months, February 1948 to February 1950, are given n Table I.

TABLE I. Raindrop sizes

Comparison of equivalent drop-diameter D_e and 50 per cent. drop-diameter D_{50} (mm)

ear	Drop-size	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
948	D_e	—	0·70	0·95	1·05	1·40	1·00	1·75	2·05	0·80	0·55	0·80	0·80
	D_{50}	—	1·20	1·20	1·35	1·40	1·35	1·25	1·30	1·35	1·40	1·30	1·30
949	D_e	0·75	0·65	0·80	1·10	1·05	1·15	2·30	1·15	1·85	0·75	0·75	0·60
	D_{50}	1·20	1·35	1·30	1·30	1·30	1·30	1·55	1·40	1·35	1·45	1·30	1·30
950	D_e	0·60	0·70										
	D_{50}	1·15	1·30										

The data obtained from the directional raingauge are shown in another way n Fig. 7, in which the rainfall incident angles for the 25 months are plotted against he corresponding wind-speeds. On the same figure are plotted the curves of ncident angle and wind-speed for raindrops with diameters of 0·5, 1·0, 2·0 and 3·0 nm, assuming the terminal velocities quoted by Best.

Another method of obtaining an indication of the raindrop size is the use of he relationship between the intensity of the rainfall and the 50 per cent. drop-size D_{50}. The 50 per cent. drop-size is defined as the size such that 50 per cent. of he volume of rain is comprised of drops smaller than this size, and Laws and Parsons ive a curve connecting D_{50} and the rainfall rate I, the equation to the curve being :

$$D_{50} = 2{\cdot}23\, I^{0{\cdot}182}$$

where D is measured in mm and I in in./hr.

The relationship applies strictly to instantaneous measurements of rainfall ntensity, and using it with rainfall intensities based on the total rainfall and duration f fall in a complete shower, or a number of showers, may not give a correct answer, ince the rainfall intensity during a shower is by no means constant. However the rror from this cause in the value of the 50 per cent. drop-size so computed will

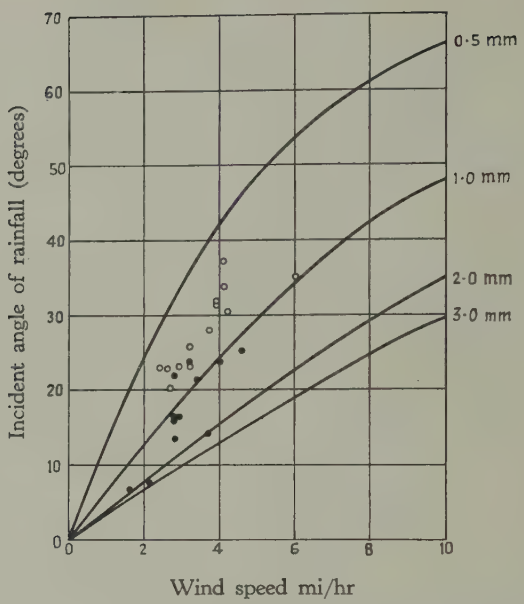

Figure 7. Variations in incident angle of rain with wind speed.

probably not in general be very great, to judge from some check computations which were made. For simplicity therefore, the overall rainfall intensities were used, and the 50 per cent. drop-diameters calculated therefrom are given in Table I.

It will be seen that in all but 3 cases the 50 per cent. raindrop-size computed from rainfall intensities is greater than the equivalent drop-size computed from incident angles. It might be expected that the two sets of values should correspond fairly closely, though the exact relationship would depend on the drop-size distribution curves. The wind speed used in the computation of the equivalent drop-sizes was that at the level of the vertical apertures of the gauge. It may be that because of the inertia of the raindrops the incident angle measured by the gauge corresponds to the wind speed at a higher level, and this wind speed would be higher, and correspond to a greater terminal velocity and therefore larger drop-size. This would account for part at least of the difference between the two values of drop-size.

8. Acknowledgment

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551.508.21

The Bellani spherical pyranometer or radiation integrator ; the calibration of an improved model, and some suggestions for further improvements

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Anti-Locust Research Centre, London

(Communicated by C. D. Ovey ; manuscript received 30 November 1949 ; in revised form 19 December 1950)

SUMMARY

The need for instruments to measure solar radiation for biological purposes has been cogently argued by Buxton but no entirely suitable instrument has been available. The radiation integrator of the Bellani type has been greatly improved by modern techniques and, with certain further suggested improvements, might become a generally useful instrument. Field calibration of the most recent model gives a mean thermal efficiency of about 75 per cent., which is much higher than that of any previous model. Of the losses, 10 per cent. is invariable and irremediable, being largely due to the glass envelope, while most of the remainder is variable and due to re-radiation from the black receiver. Suggestions are made for increasing efficiency still more and reducing variability.

1. INTRODUCTION

The Bellani spherical pyranometer was first described in 1836 as an instrument for measuring radiation. The working principle is that the radiation is converted into heat, which causes the distillation of a volatile liquid into a graduated condenser. The volume of liquid condensed gives a total or integrated value for the radiation received. The principles of the design are shown in Fig. 1.

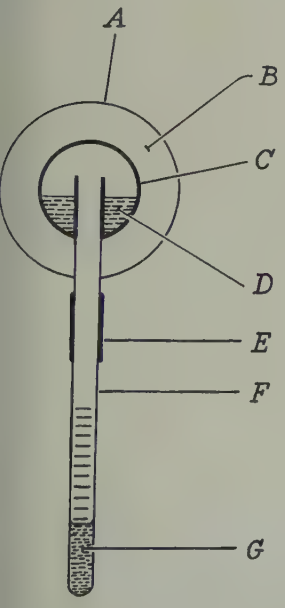

Figure 1. Diagram of the Bellani radiation integrator in improved form.

A — spherical glass envelope.
B — highly evacuated space.
C — spherical blackened copper radiation-receiver containing only the volatile liquid and its vapour.
D — volatile liquid.
E — copper condenser, with vacuum-tight seals at the ends.
F — glass burette.
G — condensed liquid in burette.

* Now with the Colonial Insecticide Research Unit, Arusha, Tanganyika.

In 1944, an improved instrument of this type was hastily designed, constructed, and calibrated in the laboratory (Gunn, *et al.*, 1945) for immediate use in connection with a study of behaviour of locusts in the field (Gunn, *et al.*, 1948). The principal improvements were to use a copper radiation-receiver rather than a glass one and to evacuate the two cavities in the apparatus to a high degree by modern high-vacuum technique. The copper receiver was spherical, coated with lamp black from a sooty flame, and sealed into the glass tube leading to the condenser to give a strong vacuum-tight joint.

As it turned out, the correlations with locust behaviour were low, so no extensive data on the integrator results were published. Since then, however, arising from recommendations of the *Commission Internationale de Météorologie* in 1935, Prohaska and Wierzejewski (1947) have discussed the mathematical theory of the instrument and described the calibration of an all-glass pattern. Our pattern has a considerably higher thermal efficiency and its readings require less correction, so it seems desirable to make some of our field data generally available and to put forward suggestions for further improvement. It is unlikely that we shall be able to develop the instrument further. The original calibrations of our instruments in the laboratory had given excessively high estimates of efficiency (Gunn, *et al.*, 1945), and we re-calibrated the instruments in the open with special precautions against reflection. The experiments were carried out on days when there was little or no cloud.

2. Calibration

A Moll pyrheliometer was used as a standard for calibration. This instrument, by Kipp and Zonen, had been calibrated by Dr. W. Mörikofer at Davos in 1936, and was very kindly lent to us by Gp.Capt. A. Walter, then Director of the British East African Meteorological Service.

It can be aimed at the sun and it measures the radiation coming from a cone of angle 20°. The readings are stated to have a mean error of $\pm$ 1·3 per cent. and a doubtful temperature correction coefficient of + 0·2 per cent. per °C.

A polished telescopic screen was devised which exposed the integrator to radiation coming only from a cone of angle 20°, *i.e.* the black sphere would receive the same radiation as the Moll pyrheliomenter. Since the integrator is a continuously-recording instrument, two rack-and-pinion movements were fitted to the screen ; these were worked by hand to keep the telescope continuously pointing at the sun.

The early experiments with this screen were done in East Africa, where a high rate of insolation was obtained, and the work was completed in Birmingham.

3. Calculation of efficiency

The calculation of efficiency is somewhat complicated, if attention is devoted to all the factors which may be involved. Prohaska & Wierzejewski (1947) have gone into the details and a simplified calculation is given here.

Let L = latent heat of vaporization of the liquid in the integrator in cal g^{-1}.
y = density of this liquid in g cm^{-3}.
s = specific heat of the liquid in cal g^{-1} $°C^{-1}$.
M = weight of copper in the black receiver in g.
c = specific heat of copper in cal g^{-1} $°C^{-1}$.
D = distillate during insolation in cm^3.
d = distillate which occurs after the period of insolation, due to the lag of the instrument.
V = volume of liquid in cm^3 left in the black receiver after the period of insolation.
t_1 = initial air temperature in °C.
t_2 = final air temperature in °C.
t = excess temperature of the black receiver over the air temperature in °C.
I = intensity of incident radiation in cal cm^{-2} min^{-1}.
A = effective exposed area of the black receiver in cm^2.
T = duration of the exposure to the radiation in min.

$$\text{Approximate efficiency (per cent.)} = \frac{100 \times \text{heat represented by distillate}}{\text{heat received as shown by Moll}}$$

$$= \frac{100\,(D + d)\,Ly}{ITA}$$

For a typical case for an alcohol-filled integrator, approximate efficiency

$$= \frac{100\,(8{\cdot}7 + 0{\cdot}4)\,221.\ 0{\cdot}79}{106{\cdot}6.\ 21{\cdot}2} = 71 \text{ per cent.}$$

(a) *Effect of changing air temperature*

Various complications arise in the more accurate estimation of the efficiency of the instrument. First, the temperature of the black sphere will not be the same after all distillation has ceased as it was before the experiment. If the air temperature rises during exposure, some of the radiation received will be used to warm the black sphere correspondingly and will not appear as distillate. The heat involved is $(t_2 - t_1)\,(Mc + Vys)$ cal. A representative value for this is $4{\cdot}25\ (45{\cdot}5 \times 0{\cdot}0909 + 30 \times 0{\cdot}80 \times 0{\cdot}566) = 75$ cal. This compares with a total heat received by the apparatus of say 1,500 cal in an hour, or 5 per cent.

The efficiency corrected for changes in air temperature is

$$\frac{100\,(D + d)\,Ly}{ITA - (t_2 - t_1)\,(Mc + Vys)\,100/90}$$

The factor 100/90 (actually $100/91 \times 100/99$) allows for the fact that the loss of efficiency is partly due to 9 per cent. reduction of radiation by absorption and reflection at the glass envelope and 1 per cent. reflection from the black sphere (see Section 4), and the quantity of heat allowed for if the factor is not used is the quantity which actually appears in the black sphere.

(b) *Effect of mean air temperature*

The values of the efficiency calculated in this way were found to be correlated with mean air temperature, with a temperature coefficient of + 0·40 per cent. ± 0·07 per cent. per °C. It was then found that the galvanometer used with the Moll had a temperature coefficient of + 0·34 per cent. ± 0·01 per cent. per °C. Consequently the Moll results were corrected to an air temperature of 15°C, using the coefficient 0·4 per cent. per °C, and the efficiencies corrected accordingly. Table I shows the crude figures and the effects of correcting for (a) variation in air temperature and (b) divergence of mean temperature from 15°C.

TABLE I. Efficiency of integrator no. 8, from 11 experiments. Column A shows the crude results ; column B gives the results after correcting for the mean air temperature, to allow for the temperature coefficient of the galvanometer of the standard instrument ; in column C allowance is also made for change of air temperature during the experiment.

Expt.	A (per cent.)	B (per cent.)	C (per cent.)
1	72·9	71·1	73·7
2	76·7	73·6	73·6
3	83·8	81·1	82·3
4	70·3	72·9	74·3
5	70·3	73·4	74·0
6	76·9	79·6	80·0
7	68·1	70·3	72·0
8	72·9	72·7	75·4
9	79·0	77·4	77·7
10	76·0	76·7	77·2
11	76·1	75·6	76·2
Averages	74·8 ± 1·35	75·0 ± 1·05	76·0 ± 0·93

4. Causes of loss in efficiency

If the causes of loss of efficiency can be traced, it is possible that variations in efficiency, as indicated in Table I, can be accounted for and removed.

(a) *Heat loss at the glass envelope*

The data supplied by Messrs. Chance Bros. indicate that at normal incidence the absorption of solar radiation in the Hysil glass of the envelope is about 1 per cent., while reflection is about 8 per cent. Consequently only 91 per cent. of the incident radiation reaches the copper receiver. Much of the radiation which reaches the black sphere does not strike the glass normally, but at angles up to about 35° to the normal ; the constants are hardly affected up to this angle, but it is clearly important for the envelope not to be smaller.

(b) *Reflection from the black surface*

This probably amounts to about 1 per cent. of the radiation incident upon the black sphere.

(c) *Pyrheliometric scale*

Dr. Mörikofer states that the pyrheliometric scale that he used in calibrating the Moll (Smithsonian 1913 scale) is the equivalent of the Ångström scale plus 3·5 per cent. ("pyrheliometric scale = Ångström scale + 3·5 per cent."). This presumably means that it is not possible to decide which scale is correct, but the Ångström scale gives lower values. It is therefore possible that 3·5 per cent. should be subtracted from the Moll values ; we have not used this correction.

(d) *Conduction and convection*

Since the envelope is very highly evacuated, convection need not be considered. The only possible route of loss of heat by conduction is along the glass tube joining the black sphere to the condenser ; a rough calculation shows that this amounts to about 0·1 per cent. only of the rate of incident radiation.

(e) *Re-radiation*

The above losses total 10 per cent. The efficiencies found (Table I) range from 72 to 82 per cent., so that 8 to 18 per cent. remains to be accounted for. Re-radiation from the black sphere must vary with the temperature of the sphere, or at any rate with its excess over surrounding temperatures, so it may account for both the reduced efficiency and the variation in efficiency. These points are considered below.

(i) *Temperature of the black sphere.* When the integrator has been exposed to the sun for a time and is then shielded from radiation, distillation continues for 10-20 min at a decreasing rate, as the black sphere cools. If the sphere and its contents are assumed to have been at a uniform temperature, that temperature can be estimated as $\frac{dyL}{Mc+Vys}$.

This equation ignores radiation losses from the sphere during residual distillation but a rough calculation shows that these losses are only about 5 per cent. of the heat represented by the residual distillate. The largest calculated excess temperature for the 11 experiments was 4·9°C and the mean was 3·5°C.

(ii) *Loss of efficiency by re-radiation.* Using Stefan's Law the radiation losses from the black sphere are approximately

$$= \sigma A\,(T_B - T_G)\,4T^3$$

where σ is Stefan's constant, and T is the mean of T_B and T_G, the absolute temperatures of the bulb and the glass. From this the losses for our instruments were 0·64 and 0·74 cal $°C^{-1}$ min^{-1} at 10° and 25°C respectively.

Assuming the glass envelope to be at air temperature, and using the mean excess temperature of the sphere calculated from the residual distillate at the end of the experiment, the estimated loss of heat from the black sphere by re-radiation averaged 2·4 cal min^{-1}, or an average of some 13 per cent. This would appear amply to explain the remaining deficiency in efficiency. Since the residual distillate and therefore the excess temperature of the sphere was found to differ in different

experiments, presumably because of different intensities of radiation, the variation in efficiency as well as its deficiency can be explained mainly by re-radiation.

5. Variation in efficiency

In the calculation of the excess temperature of the radiation receiver, it was assumed for simplicity that the black sphere and its contents were at uniform temperature ; this is only an approximation. The sphere is never quite full of liquid, and such heat as appears at the top of the sphere must be conducted down to the liquid before being transferred to it. Inevitably, the top of the sphere will be warmer. As the liquid distils away, the conduction distance increases and the top of the sphere must become warmer still. Re-radiation from the upper part of the sphere may therefore substantially increase as an experiment proceeds and, if it does, efficiency may be expected to fall progressively.

Now the instrument of Prohaska and Wierzejewski (1947) had an efficiency of about 60 per cent. when the sphere was nearly full, falling to about 30 per cent. when nearly empty. Our instrument certainly showed no such fall in efficiency as this. Fig. 2 gives the results from the four occasions in Kenya when there was continuous strong sunshine estimated with both the Moll and Integrator No. 8,

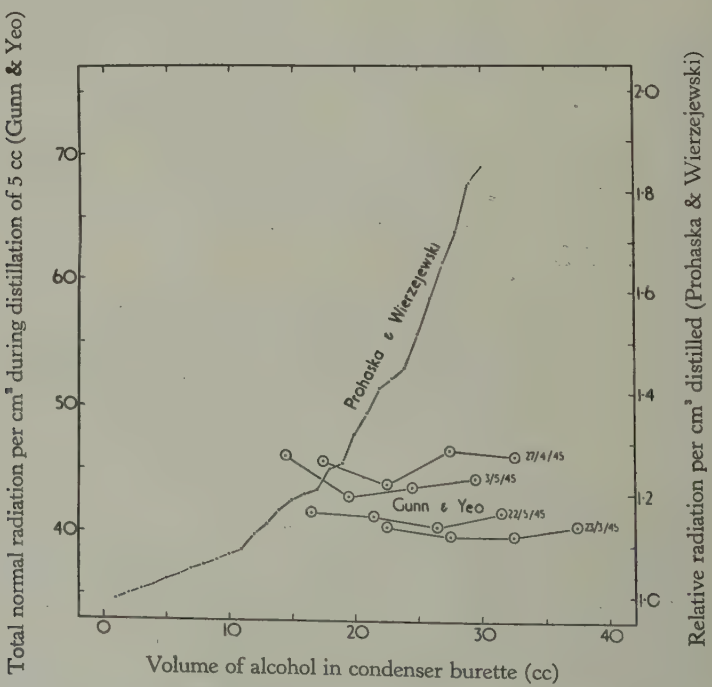

Figure 2. Changes in efficiency of radiation integrators as the alcohol distils from the spherical radiation receiver into the burette. Scale on the left : total direct solar radiation on 1 cm² at normal incidence during distillation of 5 cm³ (our data from Kenya). Scale on right : number of cm³ of distillate, with the burette nearly empty, equivalent to 1 cc at the burette readings shown (Prohaska & Wierzejewski). No corrections have been applied to the Kenya data, and reflection from the ground differed widely on the different dates.

unshielded ; in the same graph are shown data from Table I (p. 207) of Prohaska & Wierzejewski (1947). The intention is to indicate the changing amounts of radiation required to produce a given amount of distillate as the sphere empties and the burette fills. The absolute amounts of radiation cannot be given, and for our data there was certainly much reflection from the ground, which differed from day to day ; we have simply shown the total amount of solar radiation falling on 1 cm^2 exposed at right angles to the sun's rays during the distillation of 5 cm^3 of alcohol, without applying any correction whatever. Any change in efficiency between burette readings of 15 and 35 cm^3 was too small to show in our uncorrected data. On the other hand, over a similar range, Prohaska & Wierzejewski found that a given amount of distillate required over 70 per cent. more radiation when their instruments had 30 cm^3 in the burette than when it had 10 cm^3.

Although the variation in the efficiency of our instrument is so much smaller, it may still be large enough to be worth correcting, if possible ; our data are insufficient to show how large it is. The problem of reducing variation in efficiency due to re-radiation is a practical problem of keeping the rise of temperature of the sphere, especially its top, as small as possible.

6 Discussion

The field efficiencies (Table I) are about 20 per cent. lower than the earlier laboratory estimates (Gunn, *et al.*, 1945). The laboratory efficiencies should be higher than our field ones, because the laboratory intensities were lower, but the greatest cause of divergence is likely to be reflection from surrounding objects. In the laboratory, the importance of reflection was not realized and inadequate precautions were taken against this source of additional radiation.

(a) *Improving the apparatus*

It has been shown above that there are irremediable losses of efficiency of about 10 per cent. at the glass envelope and by reflection from the black sphere ; they are invariable and need cause no concern. The loss by re-radiation from the black sphere is somewhat larger and is variable ; to reduce this, the temperature of the black sphere must rise as little as possible. The sphere of our apparatus keeps as cool as it does because of the almost complete removal of air from the system, because the sphere itself is made of metal, and because of the copper condenser. It seems probable that the main defect in the instrument used by Prohaska & Wierzejewski (1947) lies in the use of a glass receiver. There seem to be two ways of improvement : to increase the rate of flow of heat by increasing the thickness of the copper wall, or to take the liquid up to the top of the sphere in some way. Thickening the copper would, of course, increase the temperature difference *through* the wall, but this must be unimportant compared with the decrease in temperature difference *along* the wall. Quadrupling the thickness of the wall to 1 mm would double the total weight of metal, including the insertion tube, to 110 g. This would increase the danger of breakage, but could be dealt with by means of struts.

The alternative of taking the liquid up to the top of the sphere has not been

investigated. A lining of blotting paper would not survive the heating of the sphere during soldering, but there may be some other way of using capillarity to keep the top of the sphere wet.

(b) *Size relations*

The decrease of efficiency as the sphere empties can be partly dealt with in another way, namely by arranging that the upper surface of the liquid shall not fall very much. With a given fall, the greatest volume of distillate can be obtained by arranging for the initial and final levels of the liquid to be at equal distances above and below the horizontal equator of the sphere.

The data are now available for designing an instrument of the appropriate size. The total radiation received depends on the square of the radius of the sphere, while the volume of liquid available depends on the cube of the radius. Doubling the radius will therefore double the time for which the apparatus will run in given circumstances. Further latitude can be obtained by choosing a liquid of appropriate latent heat. Using water instead of alcohol would allow nearly three times as long for an observation, while using ether would give a short run but a very sensitive instrument. A sphere somewhat larger than ours, with water as the liquid, would last the whole day in tropical sunshine, without a large change of level.

Prohaska & Wierzejewski (1947) suggested that the efficiency of their instrument should be reduced in some way, so as to avoid having the burette filled up before the end of the day. Clearly, a larger instrument would do.

(c) *Constructional details*

The envelope we used was a flask with the flat bottom blown out to make an approximate sphere, and this was the best possible method of construction under the conditions obtaining in 1944. The envelope should, however, be as accurate a sphere as possible and should be concentric with the spherical receiver ; otherwise reflection occurs from the inside of the envelope, acting as a concave mirror, on to the black receiver. Thickenings in the glass are to be avoided, for they act as lenses and may deflect light off or on to the black receiver.

When there is an equatorial ridge around the black sphere, the effective area exposed to the sun varies somewhat according to the sun's altitude. In order to avoid this variation, the ledge at the rim of one hemisphere, which is required to simplify soldering the two hemispheres together, should be internal rather than external.

(d) *Biological importance*

Some remarks on the use of the spherical pyranometer in relation to locusts have been made by Gunn, *et al*, (1948). The instrument gives a fairer indication than any other of the large amount of radiation which can be reflected from cloud, from the ground, and from neighbouring buildings, and so gives an idea of heating effects on solid objects. Several instruments together, each appropriately shielded, could be used to assess the relative importance of these contributions. The instrument cannot, however, evaluate the long-wave radiation given off by heated

und in virtue of its own temperature ; this seems likely to be of considerable portance in the comfort of human beings in sunny climates and also in the be- iour of other animals.

Acknowledgments

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Seasonal variation of the axes of low-latitude pressure and divergence patterns over the oceans

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SUMMARY

The seasonal fluctuations of the mean latitudes of the axes of the doldrums and sub-tropical anticyclones over all the oceans, and of the belts of maximum convergence and divergence of surface winds over the North Atlantic are evaluated and discussed. There is a lag of about two months after the sun is vertically overhead in the positions of the axes. The seasonal movement of the axes of the sub-tropical anticyclones is about half that of the axes of the other elements considered. In the North Atlantic and West Pacific Oceans the correlation between the mean latitudes of the axes of the doldrums and sub-tropical anticyclones is high.

1. THE MEAN LATITUDES OF THE AXES OF THE DOLDRUMS AND SUB-TROPICAL ANTICYCLONES

The climatic charts of the oceans prepared in the Meteorological Office include monthly charts of the mean pressure distribution over the oceans. Smoothed isobars are based on mean pressure values computed for two-degree squares in the North Atlantic and five-degree squares elsewhere. The pressure observations from which the charts have been compiled cover broadly the following periods :

North Atlantic	1921-1938
South Atlantic	1855-1899
	1921-1938
Indian Ocean	1855-1943
Eastern Pacific to 160° E. Longitude	1921-1938
Western Pacific from 160° E. Longitude	1921-1938

In order to calculate the mean latitudes of the axes of the doldrums and sub-tropical anticyclones transparent two-degree grids were placed over the mean charts and a value of pressure for each square was interpolated. The values were then meaned along two-degree belts of latitude across the oceans. The mean latitudes of the axes of the doldrums and sub-tropical anticyclones, defined as the mean latitudes of those overlapping adjacent two-degree belts whose mean pressures are a minimum and maximum respectively, were then evaluated.

The term " doldrum " is interpreted by Durst (1926) to denote the region of calms and light variable winds confined to the trough of low pressure between the NE. and SE. trades. He found that there were definite aperiodic fluctuations in the position and width of the doldrum belt which is characterized by heavy and frequent precipitation, violent thunderstorms and convergence of winds at the surface. It is a belt marked, in the mean, by vertical ascent of air on a wide scale and therefore by active divergence at upper levels.

It has been thought for some time by climatologists that the air which diverges at upper levels over the doldrum belt converges at upper levels above the sub-tropical anticyclonic belt. The theory is dynamically plausible from the point of view that the outflow of air over the doldrum belt would, in the mean, be directed polewards initially because the Coriolis force is small. As the Coriolis force increases the air would tend to turn towards the east and adjust itself more nearly to gradient wind-flow. It would be expected that a zone of convergence would occur when the more southerly (or northerly) winds from the doldrums encountered winds at higher latitudes already moving eastward under approximate gradient flow.

TABLE I. MEAN LATITUDES OF AXES OF DOLDRUMS AND SUBTROPICAL ANTICYCLONES

Month	Latitudes (degrees) of axes of Northern anticyclone	Doldrums	Southern anticyclone	Doldrums	Southern anticyclone
		Atlantic Ocean		Indian Ocean	
	(a)	(b)	(c)	(b)	(c)
J	32 N.	2 N.	29 S.	11 S.	34 S.
F	30 N.	1 S.	30 S.	10 S.	34 S.
M	28 N.	1 N.	33 S.	13 S.	32 S.
A	32 N.	4 N.	29 S.	14 N.	32 S.
M	33 N.	8 N.	28 S.	—	32 S.
J	32 N.	12 N.	26 S.	—	27 S.
J	33 N.	12 N.	28 S.	—	30 S.
A	36 N.	12 N.	28 S.	—	29 S.
S	36 N.	12 N.	29 S.	—	28 S.
O	33 N.	12 N.	30 S.	—	30 S.
N	36 N.	12 N.	28 S.	2 N.	30 S.
D	30 N.	4 N.	28 S.	0	29 S.
Mean	32·6 N.	7·5 N.	28·8 S.		30·6 S.
Range	8	13	7		7
Correlation coefficient	a-b	0·86			
	b-c	0·40			

	West Pacific Ocean			East Pacific Ocean		
	(a)	(b)	(c)	(a)	(b)	(c)
J	28 N.	14 S.	36 S.	35 N.	2 N.	32 S.
F	24 N.	10 S.	34 S.	31 N.	3 N.	35 S.
M	28 N.	9 S.	36 S.	36 N.	4 S.	34 S.
A	34 N.	3 S.	36 S.	34 N.	3 N.	32 S.
M	34 N.	1 S.	31 S.	36 N.	4 N.	32 S.
J	36 N.	7 N.	28 S.	36 N.	4 N.	28 S.
J	34 N.	8 N.	26 S.	40 N.	5 N.	25 S.
A	37 N.	7 N.	27 S.	40 N.	9 N.	32 S.
S	36 N.	10 N.	29 S.	40 N.	14 N.	32 S.
O	34 N.	2 N.	28 S.	36 N.	10 N.	32 S.
N	32 N.	0	32 S.	34 N.	6 N.	36 S.
D	30 N.	5 S.	34 S.	32 N.	4 N.	36 S.
Mean	32·3 N.	0·7 S.	31·4 S.	35·8 N.	5·0 N.	32·2 S.
Range	13	24	10	9	18	11
Correlation coefficient	a-b	0·91		a-b	0·43	
	b-c	0·82		b-c	0·16	

Table I gives the monthly-mean latitudes of the axes of the doldrums and sub-tropical anticyclones for the Atlantic, Indian, East Pacific and West Pacific Oceans and correlations between them. The correlations between the latitudes of the doldrums and the latitudes of the corresponding sub-tropical anticyclones for one month later are a little smaller than the direct correlations shown in Table I.

Fig. 1 shows the monthly fluctuations of latitude averaged over the Atlantic, East-Pacific and West-Pacific Oceans and Fig. 2 is a scatter diagram in which the mean latitude of the axis of the doldrums has been plotted against the mean latitude of the axis of the anticyclone for all cases in Table I, excluding the Indian Ocean. In the latter diagram all cases have been referred to the northern hemisphere ; thus, if the axis of the doldrums is north of the equator the point relating this axis to the axis of the southern hemisphere anticyclone has been plotted as if the two axes were reversed in hemisphere.

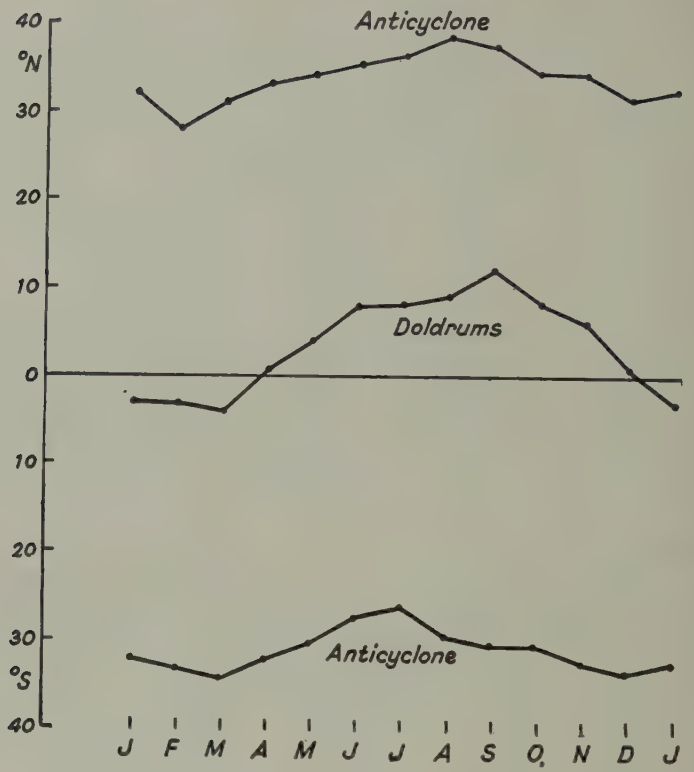

Figure 1. Seasonal fluctuations of the mean latitudes of the axes of the doldrums and subtropical anticyclones.

Figs. 1 and 2 illustrate the reality of the relation between the movements of the doldrums and sub-tropical anticyclonic belts. On the average the movement of the anticyclonic belt is about one half the movement of the doldrum belt.

An important factor that must affect the relative positions of the belts discussed is the seasonal exchange of air between continent and ocean, but this phenomenon will be discussed in a subsequent paper.

2. The mean latitudes of the belts of maximum convergence and divergence of mean vector surface winds over the north atlantic ocean

The calculation of convergence and divergence of wind over the ocean has recently received the attention of various authors ; Durst (1947), Riehl and Yeh (1950). The distribution of convergence and divergence in low latitudes and the relation of the distribution to the formation and maintenance of the semi-permanent sub-tropical anticyclones has been recognized as an important feature of the general circulation. Marine data accumulated in the Meteorological Office are particularly suitable for the investigation of such problems, using techniques of statistical and dynamical climatology based on mean data over the oceans, where conditions are less disturbed than over land masses by erratic fluctuations caused by topographical and thermodynamical influences.

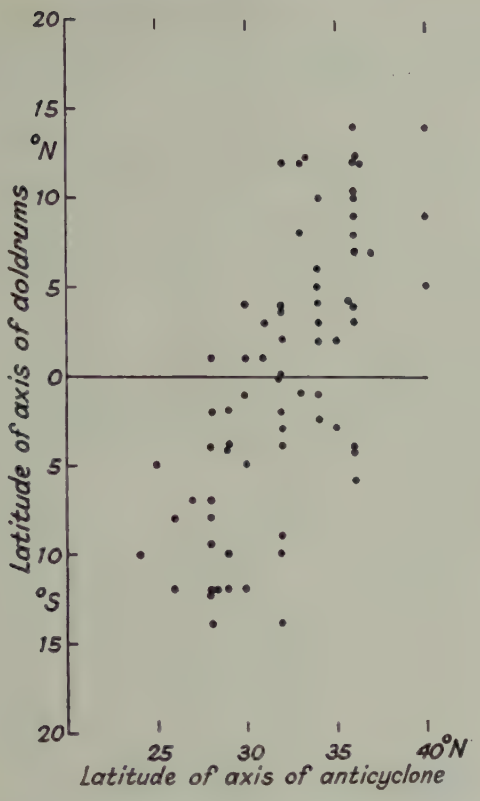

Figure 2. Relation between the latitudes of the axes of the doldrums and sub-tropical anticyclones.

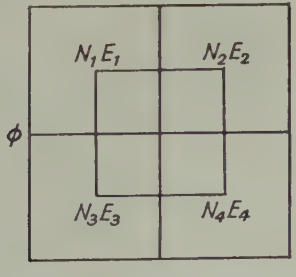

Figure 3. Calculation of convergence of mean vector winds.

The mean convergence or divergence of surface wind is proportional to the quantity

$$\pm [N_1 + N_2 - (N_3 + N_4) + \sec\phi\,(E_2 + E_4 - E_1 - E_3)]$$

here $N_1 \ldots N_4$, $E_1 \ldots E_4$ are the components of the mean vector winds of four adjacent ve-degree squares and ϕ is the mid-latitude of the squares as illustrated in Fig. 3. The alculation then refers to the central five-degree square.

The computations covered an area extending in latitude from the doldrums to th sub-tropical anticyclone and in longitude from squares adjacent to the African coast t those adjacent to the American Coast. Only squares entirely bounded by water wer considered.

The values of divergence were summed over all squares bounded by the same parallel of latitude and mean values computed for each five-degree belt of latitude from 10°S. t 40°N. latitude.

Table II gives the monthly mean latitudes of maximum convergence and divergence over the North Atlantic. The correlation between the latitudes of these belts is 0·85. The correlation between the latitude of the maximum convergence and the latitude of the axis of the sub-tropical anticyclone is 0·77. Actual values of convergence and divergence have also been given.

TABLE II. MONTHLY FLUCTUATION OF LATITUDES OF MAXIMUM MEAN CONVERGENCE AND DIVERGENCE OVER NORTH ATLANTIC (10°S.-40°N.)

Month	Latitude of maximum convergence (a)	Magnitude $\times 10^{-6}$ sec^{-1}	Latitude of maximum divergence (b)	Magnitude $\times 10^{-6}$ sec^{-1}
J	2·5	9·7	17·5	4·3
F	2·5	7·7	17·5	4·9
M	− 2·5	7·8	12·5	6·2
A	2·5	8·0	17·5	6·0
M	2·5	8·2	17·5	5·3
J	7·5	8·1	22·5	4·0
J	7·5	7·4	27·5	6·1
A	10·0	6·4	25·0	3·7
S	7·5	6·5	22·5	2·8
O	7·5	6·9	22·5	4·0
N	7·5	7·1	22·5	3·4
D	2·5	7·2	22·5	3·6
Mean	4·8		20·6	
Range	12·5		15·0	
Correlation coefficient a-b	0·85			

Table II indicates that there must be a zone of convergence at upper levels between about 15°-25°N. latitude in the North Atlantic to compensate for the divergence at the surface.

ACKNOWLEDGMENT

I am indebted to the Director of the Meteorological Office for permission to publish this note.

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551.577.21(41/2)

infall over Great Britain and Northern Ireland during the decade 1940-49

By J. GLASSPOOLE
Meteorological Office, London

(Manuscript received 25 November 1950)

In continuation of the series of maps defining the rainfall over the British Isles in h of the twelve decades 1820 to 1939 (Glasspoole 1933, 1943), a map (Fig. 1) is now en showing the rainfall for the period 1940-49. As previously, the mean annual rainfall each place has been expressed as a percentage of the average 1881-1915, but it has not n possible to continue the lines over the Irish Republic.

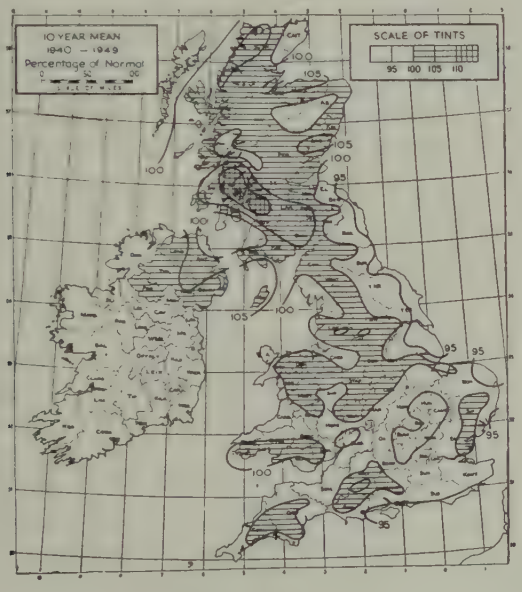

Figure 1

It will be seen from Fig. 1 that most of the country received between 95 and 105 per nt. of the 1881-1915 average. There was less than 95 per cent. along most of the E. ast from Berwick-on-Tweed to Harwich, from Reading to Huntingdon, along part of e S. coast, from Poole to Lewes, and around Gloucester. The main areas with more than 5 per cent. were around Bury and Huddersfield, to the S. and W. of Glasgow, around undee, to the S. of the Moray Firth and in W. Sutherlandshire. On the whole the smaller lues occurred in the E. and S. and the larger values in the N. and W.

The general rainfall for each year of the new decade 1940-1949 has been calculated r use in the annual volumes of *British Rainfall*, as a percentage of the average for the stan- rd period 1881-1915. The values for the decade are given below, together with the erage rainfall for 1881-1915.

TABLE I. ANNUAL GENERAL RAINFALL AS PERCENTAGE OF THE AVERAGE 1881-1915

	England	England and Wales	Scotland	Northern Ireland
Average rainfall in inches, 1881-1915	32·67	35·23	50·32	39·66
1940	101	101	98	113
1941	97	96	87	95
1942	94	94	100	113
1943	92	93	106	104
1944	100	100	105	110
1945	93	93	101	99
1946	118	118	99	109
1947	91	92	97	105
1948	104	106	123	113
1949	87	88	108	99
1940-1949	97·7	98·1	102·4	106·0

For comparison of the thirteen decades from 1820 to 1949 the general percentages are set out below.

TABLE II. DECADAL GENERAL RAINFALL AS PERCENTAGE OF THE AVERAGE 1881-1915

	England	England and Wales	Scotland	Northern Ireland
1820-1829	103·5	—	—	—
1830-1839	102·5	—	—	—
1840-1849	102·9	—	—	—
1850-1859	95·5	95·7	—	—
1860-1869	101·4	101·2	—	—
1870-1879	108·4	108·3	104·6	—
1880-1889	103·2	102·8	94·7	—
1890-1899	95·7	95·7	100·2	—
1900-1909	97·8	98·0	103·3	—
1910-1919	107·3	107·2	101·7	—
1920-1929	105·6	106·3	107·2	109·2
1930-1939	105·0	104·9	102·2	102·2
1940-1949	97·7	98·1	102·4	106·0

Over England, and over England and Wales, only the three years 1940, 1946 and 1948 gave more than the average ; over Scotland there were five wet years 1943, 1944, 1945, 1948 and 1949 ; while over Northern Ireland the seven years 1940, 1942, 1943, 1944, 1946, 1947 and 1948 each gave more than the average there.

Over England the deficiency from average during the decade 1940-1949 was small, being only 2·3 per cent. or 0·7 in. per year. Nevertheless it was the driest decade since that of 1890-99, with a mean of 95·7 per cent., and gave less than the mean of the three decades 1910-1939 by as much as 8·3 per cent. or 2·7 in. per year. The decade 1940-1949 was the third driest in the series of 13 decades for which comparable data are available. Over Scotland, on the other hand, 1940-1949 ranks as the fourth wettest decade in the eight for which records are available.

The features of the distributions shown on the decadal maps must reflect the peculiarities of the distribution in the standard period 1881-1915. When averages are available for the next standard period 1916-1950, it will be possible to use 70 years, 1881-1950, as the standard of reference and so review the changes from decade to decade from a broader aspect.

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Correspondence

Large-scale vortex motion in the atmosphere

If a short rejoinder to Mr. James (*Q.J.*, **77**, p. 137) may be permitted I would like to put the following points :

1. With regard to the Rankine vortex, Mr. James is now shifting his ground. He now appeals, quite rightly, to the diffuseness of the Rankine vortex as an argument against its use in meteorology, but this is not the argument put forward in his paper and it was with his original paper that I was concerned.

2. Mr. James asserts that a few thousand kilometres is infinity for all practical purposes but has not yet defended the limitation on his function $f(s)$ (*Q.J.*, **76**, p. 261), for the argument used in the paper for this limitation depends essentially on carrying out the integration over the infinite range. If in Eq. (12) of Mr. James's paper the upper limit, ∞, is replaced by X, then any well behaved function $f(s)$ will do. Again it is a question of diffuseness that limits $f(s)$, and $f(s) = \exp(-s^2)$ is chosen for rapid convergence to the value of the integral for smallish values of s, and not because it happens to decrease more rapidly than $1/s^2$.

3. There is a basis for my assertion that $R(z)_{max}$ is the parameter to use, for it is simply Mr. James's R *by his definition on p. 257*, where there is no mention that R is measured at the surface. I have also given a reference showing that R, as defined by Mr. James, need not necessarily occur at the surface.

4. Mr James misconceives the function of a critic if he thinks that a critic must produce something better. Turning from the obvious examples of literary and dramatic critics, I would criticise a proof that the circle can be squared but I could not produce a better proof. What I could show is that the squaring cannot be done, and in the same way I believe that weather systems show that simple vortices are inadequate to describe the motion.

5. The implied assumption in Mr. James's reply is that analysis is in terms of vortices or nothing. But Mr. James is as aware as anyone that analysis in terms of vortices is by no means the only way to attack the problem, and that there is a great deal of difference in admitting vorticity and attempting to interpret it as a system of simple vortices. (*e.g.* see Sutcliffe, *Q.J.* **73**, p. 370).

Finally I would recall Maxwell's burlesque of *Comin' thro' the rye* and suggest that the lines " Ilka body has its measure, ne'er a ane hae I, But a' the lads, they measure me, Or at least, they try ! " are applicable to vortices.

E. Knighting

Luton.
24 January 1951

Reply

Mr. Knighting's rejoinder goes far towards defining the essential divergence of viewpoint between us. I am challenged to defend my restriction on $f(s)$ that it should tend to zero more rapidly than $1/s^2$ as s tends to infinity. This is the condition that $\int_0^\infty f(s)ds$ should exist (f is restricted to monotonic functions) and be finite. If $f \equiv 0$ for $s > X$ my condition is automatically satisfied ; f does not have to tend to zero, it is already there.

My restriction merely states that if f extends over an infinite range it must satisfy certain conditions for large s. This *in no way* excludes the use of functions confined to a restricted range of s ; indeed, I consider such a function, the sinoidal vortex, in my paper. For generality, we should always take infinity as an upper limit unless there is some well-defined physical limitation to the range. This is a convention understood by all mathematicians, and even a meteorologist uses it, as for example, when he integrates vertically to infinite height. The atmosphere is not *really* of infinite vertical extent, and the symbol ∞ in physics can only be taken to mean " far away as no matter."

Mr. Knighting's point 3 obviously rests on a misconception, for which obscurity in the original paper may be partly to blame. Therefore, it should be stated now that the parameters h and R refer to a horizontal cross section of the vortex. A full three-dimensional representation results when we specify how these parameters vary with height. Casual experience with synoptic charts reveals that the scale of a vortex does not vary perceptibly with height, so that to a first approximation the surface R may be taken to represent R at all levels. The data used in my paper related essentially to the surface.

With these red herrings successfully swallowed, we may now turn to the essential divergence of viewpoint between Mr. Knighting and myself. I do not think I misrepresent Mr. Knighting in referring to him as an adherent of the pure field theory. The field of pressure, of wind, divergence, or what not, is regarded as the essential reality, the *Ding-ansich*. Any analysis, such as mine into flow field and vortex is invalid, a resolution of the irresolvable. A vortex can have no identity or structure of itself, it is inextricably part of the whole.

I must confess that I see little prospect for the advancement of our science along these lines, nor have the field partisans produced anything to increase one's confidence in the essential correctness of the approach. Your pure field theorist displays an inordinate liking for such entities as divergence or vorticity which, if measurable at all, press very closely against the limits of experimental error. In that twilight land where truth and fancy meet, the faithful may draw their holy isopleths in a fashion fitting to the current canon, but they must be prepared to convince none but the faithful.

The alternative viewpoint is based on the assumption that vortices are entities with a separate existence, of definable structure, and exist embedded in a general flow field. The sole test of such an approach is to attempt to measure these entities, and to convince critics that one has succeeded in measuring what one set out to measure.

It would seem that we ' vorticists ' have the average forecaster's support. The forecaster does not speak of an ' interesting ' configuration in today's pressure pattern, but of a depression ' centred West of Ireland.' By merely assigning a name and co-ordinates to the phenomenon he endows it with a certain integrity and being. He may be wrong, of course, but his error bespeaks at least an arguable heresy.

Goldie (*Geophys. Mem.*, Nos. 72 and 79) has published velocity profiles of a number of depressions, strongly suggesting a broad-scale structure common to all depressions. If these are not the profiles of wind in a vortex, can Mr. Knighting tell us what they are ? If a depression considered as a distinct entity is an illusion, can the simon-pure field theorist explain to the non-initiate how this epi-phenomenon gives rise to the illusion of structure ?

Finally, I should like to ask whether Mr. Knighting is willing to extend the pure field theory to a tropical low. Is it suggested that such a phenomenon is a rather interesting configuration in the pressure field that sometimes pops up in tropical regions ? Or is the isobaric pattern symptomatic of a vortex distinct in itself, and measurable ? (see James, *J. Met.*, February 1951).

If one accords a measure of reality and measurability to the vortex structure in the tropics, why not also in temperate regions ?

R. W. James

Melbourne, Australia.
13 February 1951

551.508.21 : 551.521

Notes on the measurement and estimation of atmospheric radiation — 2

By G. D. ROBINSON

(Read 22 November 1950. See *Q.J.*, **76**, p. 37)

Discussion

Asst. Prof. P. A. Sheppard : Dr. Robinson has warned us not to trust the radiation chart when applied to the evaluation of radiative flux in the middle and upper troposphere. But we are very anxious to apply it to just those levels ! I wonder therefore whether he has considered checking the chart against the only observations that I know of the divergence of radiation in the free atmosphere, namely those of Ångström (*Beitr. Phys. frei. Atmos.*, **14**, 1928) made in a manned-balloon flight up to 4 km.

On the question of the temperature dependence of emissivity, there are two effects to be considered — the effect of the variation of energy *distribution* with variation in temperature (relatively less energy in short wavelengths as temperature decreases), and the effect of temperature on the structure of individual absorption " lines." The classical theory for the latter suggests emissivity $\propto T^{-1.4}$ in those path lengths u for which emissivity $\propto u^{1/2}$ while Cowling has suggested that the variation may be as $T^{-0.35}$. Do the present observations throw any light on these variations ?

Mr. E. Gold : The assumption that emissivity is independent of temperature is equivalent to assuming atmospheric radiation proportional to T^4. But as total black-body radiation varies as T^4 while radiation near the maximum varies as T^5 the radiation from water vapour must vary as a power T^n where n is less than 4. Thus, if n is 3·5, E for a water-vapour atmosphere must vary as $T^{-1/2}$ on this account.

Sir David Brunt : I wish to explain that my statement that the total radiation in the water-vapour bands is proportional to $T^{3.5}$ is based upon the figures given for black-body radiation in these wavelengths, in the Smithsonian Physical Tables. When plotted on logarithmic paper, they gave an index of 3·49. I would further ask Dr. Robinson whether he has taken account of latent heat liberated by condensation, and whether this was appreciable.

Dr. R. C. Sutcliffe : Dr. Robinson's conclusion that, on a radiation night, the air in the lowest metre of air is cooled by radiative interchange at a rate ten times greater than the measured total cooling, so implying that warming by turbulent diffusion of heat from above must provide a near-balance, might have very important implications in the fog problem. It would be interesting to know to what height above the surface this state of affairs extends ; does direct cooling by radiation remain the important factor through 10 m or 100 m, the fog-forming layer ?

It appears that the temperature profile and the variation of eddy diffusivity with height become such that turbulence is actually a warming process in the lowest levels. Since the same process is responsible for the diffusion of moisture, which deposits as dew on the ground is it possible that with an inversion of the hydrolapse diffusion actually tends to an increase of humidity in the lowest levels. One might at least be prepared to find that with an inversion of the hydrolapse above there would be no further net loss of moisture from the surface layers and, with cooling maintained by radiation, this condition might be critical for fog formation.

For condensation to begin in the free atmosphere with an inversion of temperature an inversion of the hydrolapse is necessary. Dr. Robinson's results suggest that it might turn out to be almost a sufficient criterion.

Dr. G. D. Robinson (*in reply, partly communicated*) : I can make no very useful comment on the remarks of Sir David Brunt and Mr. Gold, having much more confidence in the accuracy of the measurements than of my interpretation. Professor Cowling has said elsewhere that the suggested increase of emissivity of water vapour with increasing temperature is quite inexplicable on present theories. It may, however, be significant that the atmospheric problem concerns water in the presence of carbon dioxide, which effectively takes out almost 20 per cent. of black-body radiation *near the intensity maximum*. This, and the use of the most recent values of the absorption coefficients of water vapour might affect considerably the relation expressing that part of the dependence of emissivity on temperature which is due to the changing black-body energy distribution. Possible alternative explanations of the observations, which I have attempted to investigate, are : (*a*) the inadequacy of the pressure " corrections " applied to the radiating path lengths and (*b*) changes in the emissivity of CO_2 rather than that of water vapour.

I have not attempted to investigate Ångström's results. Measurements obtained on the ground by numerous observers at the time of Ångström's work do not seem precise enough for the purpose, and measurements in the air would be rendered still more doubtful by the increasing importance of the longer wave lengths, at which the absorptive properties of the receiver surfaces used in these experiments are not known.

To reply at any length to Dr. Sutcliffe would lead the discussion far from its main subject. The measurements and computations of the change in radiative flux in the lowest metre described in the paper refer to cloudless nights with moderate winds. Similar investigations on " calm " nights (made in collaboration with Mr. N. E. Rider) show that in the conditions obtaining immediately before the formation of radiation fog the radiative cooling becomes very small indeed. In the course of the same work many instances of the occurrence of an inversion of hydrolapse in the lowest layers, without subsequent fog formation, have been observed. An account of this work, which has been prepared for publication, also contains a discussion of the heat liberated by condensation.

551.573 : 551.582.2(41,42)

Evaporation over the British Isles

By H. L. PENMAN

(Read 22 November 1950. See *Q.J.*, **76**, p. 372)

Discussion

Mr. E. Gold : I want to say a few words about Thames valley ' run-off.' It is relevant to evaporation because broadly speaking the rain which is not evaporated and carried away in the air all runs off — in time. Strictly the run-off for the Thames valley should include the milk and bacon and vegetables carried to London and the sea-side towns. I mention " carried away " because in a large area some of the evaporated water is returned to the area as precipitation (rain or dew) without leaving the area. The allowance for this in the Thames valley would be small but in the basin of the Amazon or Mississippi it would be substantial. The formula commonly quoted for connection of rainfall and run-off is $F = 2R/3 - 9{\cdot}7$ (inches) where F is annual run-off and R is annual rainfall. If the rainfall were less than 14·5 in., F would be negative and the water would presumably run back up the rivers — a conclusion which fortified my feeling that a better formula could be devised. Dr. Glasspoole kindly provided me with just over 40 years' values of rainfall

and run-off for winter (Oct. – Mar.) and summer (Apr. – Sept.) for the Thames valley. The values of run-off for individual years, computed from the formula derived from these data, gave better agreement with the recorded values ; but there were still some large differences especially in years with large rainfall in the preceding summer, even after allowance in the formula for the average effect of the preceding summer rainfall. Accordingly the rainfall for each season was divided into two parts, one for each quarter, by reference to the monthly values, and revised formulae obtained by using these quarterly values.

The revised formulae are :

$$F_w = \cdot 32R_1 + \cdot 31R_2 + \cdot 29R'_4$$
$$F_s = \cdot 18R_2 + \cdot 22R_3 + \cdot 10R_4$$
$$F = \cdot 32R_1 + \cdot 49R_2 + \cdot 22R_3 + \cdot 29R'_4$$

where F, F_w, F_s are the annual, winter and summer run-offs respectively.

R_1, R_2, R_3, R_4 are the rainfall amounts for the four quarters Oct.-Dec., Jan.-Mar., Apr.-June, July-Sept. respectively and R'_4 is the rainfall for the quarter July-Sept. immediately preceding the winter and the year to which F_w and F refer. R_2 of course refers to the quarter Jan.-Mar. preceding the summer to which F_s refers.

These formulae give values fitting more closely the recorded amounts but there were 7 years out of 42 with differences over 3 in., the largest being 3·7 in. in 1934-5. With the formula $2R/3 - 9{\cdot}7$, there were 13 years with differences over 3 in., the largest being 6·8 in. in 1927-8. In 3 of the 7 years mentioned the run-off exceeded the computed values. These years were 1924-5, 1927-8, 1946-7. The year 1924-5 followed an exceptionally wet summer and although the rainfall in the quarter Apr.-June normally has no effect on the run-off of the following winter, it may do so when the succeeding quarter is also very wet. The year 1927-8 followed the wettest July-Sept. quarter of the whole period and although the formula makes allowance for the rainfall of the preceding July-Sept. quarter, it is unlikely that the allowance is sufficient for an exceptionally wet quarter. In the winter 1946-7 there was exceptional snowfall and the run-off in March was greater than for any month in the whole period.

An outstanding feature is the large contribution of the quarter July-Sept. The rainfall of this quarter contributes more, proportionally, to run-off than the rainfall of the quarter Oct.-Dec.

Communicated later : Still better formulae have been derived by relating the difference of F from its average value to the differences of R_1, R_2, *etc.*, from their average values.

Capt. W. N. McClean : There does not seem to be enough importance given to run-off, in the balance sheet of water on a catchment area. The paper gives the impression that the rainfall on an area is a known quantity, and that estimated evaporation could be deducted therefrom and provide a value of run-off for the engineer. The engineer however has to deal with actual run-off ; and it has to be " measured " and not " mathematical."

In a recent book, *Applied Hydrology* by R. K. Linsley Jnr., M. A. Kohler, and J. H. Paulhus (McGraw-Hill), it is pointed out that stream flow is the only portion of the hydrologic cycle in which moisture is so confined as to make possible reasonably accurate measurement of the volume involved. The difficulty lies in the accuracy of that measurement and in the continuous record of water level to which flow measurements have to be related. Trained survey staff and the best equipment for measurement and record are necessary. Then, at a principal gauging station, you have the measured run-off, month-by-month or year-by-year, for correlation with rainfall during the same month or year throughout a long record. The relative losses of the seasons (evaporation and absorption) are obtained ; and, if the rainfall on the area is correct, there is a figure of loss of 19 in. in the year on the Thames, or the 14 in. of the earlier years of water-supply engineering.

The continuous record of a river's stage and flow, day-by-day or hour-by-hour, at a principal gauging station, retains a sequence of events which is lost in monthly correlations ; and the graphical record of aggregate rainfall and run-off supplies all the data for the assessment of storages and loss.

For intermediate sites on a catchment area, where there is no measurement or record of run-off, the rainfall minus run-off of the paper will be known from the principal stations and will provide a value of run-off for the intermediate station.

Dr. J. Glasspoole showed two slides taken from a paper, *Rainfall, run-off and evaporation*, recently published by the British Waterworks Association. They emphasized the necessity for taking rainfall and run-off records averaged over a long period and for considering the difference in the storage in the river, streams and ground, at the beginning and end of the period, in order to get the best values for the loss.

In the Thames valley above Teddington the loss in a year of average rainfall could vary by 10 or 11 in., and in any group of 10 consecutive years by 4 in. The difference in the run-off during years of average rainfall could be explained by reference to differences in the seasonal rainfall of that year and earlier years, involving differences in ground storage.

It was also pointed out that the long-term trend in the loss, as shown by running 10-year means, was quite distinct from that of either the rainfall or the run-off ; the peaks occurring earlier with the loss curve. This suggested that when a run of wet years set in, a factor of some importance was the amount of rain used in making good ground storage.

Mr. L. C. W. Bonacina : I think Dr. Penman is much to be congratulated on so successfully negotiating the refractory problem of evaporation by circuitous means, but he does not explain in the paper why E_0 cannot be determined directly with evaporation tanks. Is it because the figures depend on size and shape of tank or too much upon its immediate surroundings — more than is the case with raingauges ?

Dr. Penman's evaporation map of the British Isles is a little disillusioning as regards SE. England where evidently the margin of safety is much smaller than we had thought. In the rain-shadow estuary of the Thames the annual evaporation is practically equal to the small annual rainfall of 20 in. and this throws a good deal of light on our uncomfortable experience of having to contend with a more or less serious drought every third or fourth summer.

It is likely that the map will be used much by water engineers and others in the coming years which are sure to bring a succession of water crises, not because the rainfall is going to decrease progressively but because of the progressive increase in the demands we shall make upon it. Look at the increased use of water in the last half century. The Victorians managed to achieve immaculacy on a much more restricted use of water than is judged to be necessary in the brave new world of today. Thus the question arises whether our climate will be able to afford the unlimited use of water indefinitely. I do not think it will unless either we store the surplus water of wet seasons against inevitable droughts, or else draw water far more extensively than at present from the rainy mountains of the West. The latter course besides being very costly would have the pernicious effect of ruining the scenery by diverting so many mountain torrents into pipes and aqueducts. The artificialising of the hydrology of the country has already gone far enough. Around London what with the constant shrinking of the water table in the chalk, the erection of houses, and the tapping of head-springs, there is hardly a stream which does not rise lower down its valley than it did 50 years ago, leaving a chain of pools where formerly there was a good flow.

So altogether I think that Dr. Penman's work will bear in many ways on the complex water problems which will beset our country in coming years and I hope he will continue it.

Dr. H. L. Penman (*in reply*) : The problems raised in discussion are largely philosophical, but Mr. Gold introduces two matters of fact. Thames-valley farming would be very good indeed if it produced an average annual yield of 20 tons per acre, but even if we assume that this is all water and is all removed the equivalent annual water loss is only 0·2 in. As the annual evaporation loss is about 20 in. the produce can be ignored. Nor does it matter how many times the same molecules of water pass across the earth-air boundary : if the amounts moving vertically are correctly measured or estimated the excess of " down " over " up " must give the amount that slips out of the system sideways.

If we accept Capt. McClean's statement that run-off is the only hydrologic quantity that can be measured with reasonable accuracy, must it always remain so ? I don't think

the statement is true : it may be easier to get a run-off figure than a rainfall figure, but ease and accuracy are not synonomous, and neither is necessarily a measure of importance. My object was to show that the third main component in the water balance sheet could be estimated from physical principles. The route may be circuitous as Mr. Bonacina says, but it is clear (an advantage in looking for errors), and it is purposeful because it keeps separate, to some extent, the roles played by weather, by vegetation and by soil in the evaporation process. Capt. McClean, or his fellow engineers who have lived with a river or river system throughout their professional lives, can use a single observation and memory of recent weather to make a reasonably accurate prediction of river behaviour for almost any assumed weather in the immediate future. As a check, and an improvement on impression, they might turn to a graphical record, or to a formula such as that worked out by Mr. Gold for the Thames, but, as Mr. Gold himself points out, even with five terms in the regression equation there are still periods in which the estimated run-off is seriously in error. Statistically the new relation is better than the old but we are little nearer getting at reasons. Dr. Glasspoole helps here in emphasizing for this particular case what I stated generally, that the water balance must take into account changes in ground storage and drying in the root zone of the soil. As I have shown elsewhere (*J. Inst. Water Eng.*, 1950, **4**, p. 457) allowance for these factors combined with the physics of the evaporation process can give an adequate account of the month-to-month variations in performance of a river over a long period of years. So even on a British river system about which so much is known the physicist's approach – despite its present imperfect state — can add to engineers' knowledge, and increase understanding of existing knowledge. For some parts of Britain, and many parts of the world, there is no hydrologic information available but there are meteorological data from which evaporation can be estimated : for one such area in England local estimates range from 5 to 25 in. per annum, whereas from the map it should be about 16 in. per annum. The general agreement elsewhere between the theoretical and direct estimates encourages belief that the true value for this area is not very far from 16 in. per annum.

Mr. Bonacina's kind remarks contain one direct and one indirect question. (1) Why not use evaporation tanks ? — (*a*) The paper is testing a physical theory ; (*b*) exposure, rim height, surround, colour of walls and cleanliness of water all affect tank readings. (Clean water in a light-walled tank is highly undesirable.) (*2*) Can we afford unlimited use of water ? — Probably not, but it should be remembered that our water problems alternate between having too little and having too much, and some of the problems of control within space and time are not insoluble. Further, the problem of the falling water-table may be more geological than social : there is evidence suggesting that it has been going on since the Roman occupation 2,000 years ago.

551.511 : 551.551 : 551.465

The application of the similarity theory of turbulence to atmospheric diffusion

By G. K. BATCHELOR

(Read 20 December 1950. See *Q.J.*, **76**, p. 133)

Discussion

Mr. H. Charnock described briefly the analysis to which Dr. Batchelor referred on p. 143 of his paper. The results were inconclusive but encouraging ; a fuller account of them has been submitted to the Society for publication.

Dr. R. S. Scorer : I would like to ask Dr. Batchelor where he expects to find in the atmosphere, turbulence of a kind to which the theory can be applied. (Dr. Batchelor : everywhere). The difficulty is that we seldom have much idea how the turbulence is created, and until we know that we do not know in what part of the spectrum most of the turbulent energy is being created and we cannot apply the similarity theory.

An example of considerable interest is the clear-air turbulence recently investigated by Dr. G. S. Hislop for British European Airways. It seems possible that this may be due to waves of length within a certain range being unstable, producing the turbulent effect because no single wavelength predominates. In this case, and indeed in most turbulence surely, however it is created, the predominant eddies will be those of the size that is being created, and the theory will be inapplicable.

Mr. N. E. Davis : The practical application of the similarity theory to the observations of Durst and of Richardson and Stommel is related to the expression $[\overline{l(t)} - l_0]^2$ whereas the theory uses the expression $\overline{l^2(t)} - l_0^2$. These two expressions are not the same unless $\overline{l(t)} = l_0$. This is not the case as may be shown by a simple graphical argument. In fact $\overline{l(t)} > l_0$. This may to some extent explain the difficulties in the practical results.

Dr. E. T. Eady : I congratulate Dr. Batchelor on his scholarly presentation of the similarity theory of turbulence initiated by Kolmogoroff. The argument which derives the consequences of statistical " de-coupling " between the primary turbulence-producing processes and the secondary small-scale features of the turbulence appears to be sound but does it get us very far ? In meteorology and climatology we are concerned principally with the transfer properties of the turbulence, determined mainly by the large-scale primary processes to which the similarity theory does not pretend to apply. It is the great virtue of similarity theories that no knowledge of the mechanism is involved and we do not have to assume anything about the nature of "eddies" ; anything which has "size" (such as a Fourier component) will do in our description of the motion. But this emptiness of content is also their weakness and they give us very limited insight. It is true that a similarity theory which could be applied to the *primary* turbulence-producing processes would be of great value but there is no reason to expect that anything simple can be found : when several non-dimensional parameters can be formed, similarity theory, by itself, cannot do much.

Similarity theories are attractive to those who follow Sir Geoffrey Taylor in rejecting crude hypotheses regarding "eddies," mixing-lengths, *etc.* But those who try to determine the properties of turbulence without such (admittedly unsatisfactory) concepts must show that they have sufficient material (in the shape of equations) to determine the answers. If this is not the case it will be necessary to develop some new principle in addition to the equations of motion and the nature of this principle may be brought to light in a study of the mechanism of the primary turbulence-producing process *i.e.* by trying to refine or modify what we mean by an " eddy " rather than by completely rejecting the concept.

Dr. E. C. S. Megaw : During the past year or two I have been interested in a different aspect of atmospheric turbulence in which the similarity theory has been found to have interesting possibilities of practical application. The small fluctuations of refractive index which arise from turbulent motion of the air provide the explanation of one well-known physical effect — the twinkling of the stars — and have recently been found to play an important part in the propagation of the shortest radio waves to long distances. A preliminary account of some work in this field is in course of publication (1950 *Nature*, **166**, p. 1100). It may be of interest to note that the astronomical observations of stellar scintillation appear to be consistent with the spectrum predicted by the similarity theory ; and that on this basis an approximate theory of the scattering of short radio waves in the troposphere has given numerical results which are in reasonable agreement with recent experiments. While there are here, as in the diffusion problems which Dr. Batchelor has discussed, some practically important aspects which are outside the scope of the principles of similarity, I believe their usefulness in the study of atmospheric turbulence may prove greater than would at first sight seem probable. It does not seem too optimistic to suggest

at there is a field of common interest to meteorology, aerodynamics, observational tronomy, and radio in which the first steps have already been taken towards a synthesis which each of these sciences may contribute something of benefit to the others.

Dr. G. K. Batchelor (*in reply*) : I quite agree with Drs. Scorer and Eady that the milarity theory first put forward by Kolmogoroff tells us nothing about the large-scale mponents which contain most of the energy of the motion. Unfortunately, most of the nportant transfer properties of the turbulence are determined by these large-scale com-nents, so that the practical usefulness of the theory I have described is severely limited. rhaps, to avoid misunderstanding, I should have entitled my paper " The application the similarity theory of turbulence to *relative* diffusion in the atmosphere," since the eory applies only to relative motions which are dominated by the small-scale components the turbulence. But I still think there is considerable cause for satisfaction in having hieved an understanding of part — albeit not the most important part — of the mechanics turbulent motion. When all is dark, a little light is very welcome. And moreover we ay yet find — as Dr. Megaw's contribution emphasizes — that there are several interest-g natural phenomena which are governed by the small-scale components of the turbulence d which come within the scope of the similarity theory.

Mr. Davis is quite correct in pointing out that $\overline{[l(t) - l_0]^2}$ is not equal to $\overline{l^2(t)} - l_0^2$. owever it is true that $\overline{[\xi(t) - \xi_0]^2}$ is equal to $\overline{\xi^2(t)} - \xi_0^2$, where $\xi(t)$ is one component of e vector whose magnitude is $l(t)$, and the measurements made by Richardson and Stommel ere of $\xi(t)$, as explained on p. 144 of my paper. The observations mentioned in my paper e very far from being suitable as a test of the validity of the theory.

551.509.317 : 551.524.7 : 551.547.519.2

Advective temperature change in the 1000-700 mb and 700-500 layers

By J. M. CRADDOCK

(Read 20 December 1950. See *Q.J.*, **77**, p. 51)

Discussion

Dr. E. T. Eady : The regression coefficient between advective temperature change nd actual temperature change obtained by Mr. Craddock is intelligible on the basis of dynamical theory. I have shown elsewhere that disturbances grow most rapidly when subject of course to boundary constraints) the air particles are displaced in a direction vhich has half the slope of the isentropic surfaces (when the air is unsaturated) and the esulting temperature changes are only half those corresponding to horizontal advection. A similar final result is obtained when the air is saturated. Applying the principle that the disturbances which grow fastest are the dominant ones we should expect displacements f this kind to predominate.

Mr. H. Charnock : The technique of interpolation described in the appendix of Mr. Craddock's paper should be compared with that described by Wiener in his well-known but mathematically formidable *Interpolation, extrapolation and smoothing of stationary time series*, 1949 (Chapman & Hall).

The similarity theory would not be expected to apply to Mr. Craddock's observations et the lag-correlations he computes are roughly of the form the theory predicts. There

is other evidence that the theory applies, at least roughly, in circumstances other than those described in Dr. G. K. Batchelor's paper (*Q.J.*, **76**, p. 133).

Dr. J. M. Stagg : If we can regard the casual connection between two quantities as being represented roughly by the square of their correlation, we can infer from Mr. Craddock's figures that advection accounts on the average for only about one third of the thickness changes. But there must be many synoptic situations when advection plays a greater part than this, otherwise it would surely be impracticable in the present state of knowledge about the dynamic and non-adiabatic contributions to make much use of thickness patterns in forecasting. Should not Mr. Craddock give some indication of how the closeness of correlation between thickness tendency and advection varies in different meteorological situations ?

Mr. D. G. Harley : I was surprised by the small proportions of cases where Mr. Craddock thought the movement of thickness lines was almost entirely advective. Behind cold fronts (which are not infrequent) the thickness lines close to the front normally move with the speed of the front. However, these cases certainly are mainly in the lowest layers of 1000-700 mb and Mr. Craddock was discussing higher levels as well. There one could expect thickness lines to move more slowly than advection ; for, considering the frequent case of a large-scale ridge and trough system, in which the winds at high levels are much stronger than those in the same system at low levels, thickness lines high up which are associated with frontal waves would move away from these waves, by pure advection. That they do not, indicates the descent of air at the rear and ascent at the front of the wave which we expect for other reasons to occur. The correlation figures for the 700-500 mb layer confirm that dynamical (vertical-motion) effects are greater there than at 1000-700 mb.

Some while back we hoped that, though vertical motion produced the rain, we need not really worry about it much otherwise. Now it has become increasingly clear in recent years how important these motions are.

Mr. J. M. Craddock (*in reply*) : In reply to Dr. Stagg, it would indeed be valuable to know the correlation between thickness tendency and advective component in various meteorological situations ; the present work, which treats all situations together, is intended only as a starting point. In the meantime, there is a good deal of qualitative knowledge, or surmise, on the way the processes interact in particular situations. A synoptic chart on which there are three or four well defined pressure systems could be divided into perhaps 6 to 12 regions, related to the pressure systems and to geographical features, in each of which there is a definite type of thermal balance.

For example, in an open warm sector there is usually a region where advective warming outweighs dynamic and non-adiabatic cooling, while there is a very typical region ahead of the warm front of an occluding depression, in which advective warming is outweighed, apparently by dynamic cooling, leading to the rapid weakening of the upper warm tongue. The identification of such regions is a useful aid in estimating the instantaneous manner in which the thickness is changing, but its main practical value is in upper-air analysis, improving the consistency of analysis at six-hourly intervals. In forecasting for 24 hours ahead, particularly when drawing a forecast upper-air chart (*prontour*) to agree with a forecast surface chart (*prebaratic*), the upper-air forecaster has to rely on more general ideas. Perhaps his most valuable tool is the empirical fact that the changes in thickness pattern associated with a significant pressure system are about as characteristic as the pressure changes themselves, so that he can estimate the thickness changes which correspond to the degree of pressure development shown on the surface prebaratic. In reply to Mr. Harley, cold air moving south behind a cold front over cold land often presents the nearest approach to purely advective thermal change we are likely to experience. This occurrence is more frequent over Eastern Europe or central North America than over England and it may be that the correlations in continental regions between thickness tendency and advective component would differ from the values found here. The latter relate to all synoptic situations, taken as they come, in a period of six months, and, I think, are probably typical of maritime and coastal regions in general.

Proceedings at meetings of the Society

Popular Lecture, 4 December 1950

Dr. J. M. Stagg, O.B.E., being in the chair, the President, Sir Robert Watson-Watt, B., F.R.S., gave a popular lecture on "Wireless in weather watching" at the Royal stitution. The lecture was open to sixth-form students of whom some 250 were present, arrangement with the education authorities of the London, Middlesex and Surrey County uncils.

Joint Meeting with Royal Aeronautical Society, 14 December 1950

A joint meeting with the Royal Aeronautical Society was held at the Institute of Civil ngineers. Dr. G. S. Hislop read a paper on "Investigation of gusts," describing work ne on the atmospheric phenomenon of clear-air bumpiness. The Society was represented the platform by Asst. Professor P. A. Sheppard. Several members took part in the suing discussion.

Special General Meetings, 13 December 1950

Dr. R. C. Sutcliffe, O.B.E., being in the chair, two Special General Meetings, following e another, were held.

At the first meeting consideration was given to the draft of the new by-laws, as recommended by Council, and previously circulated to all Fellows. After a very full discussion e following resolution from Council was proposed from the Chair :

"That the new by-laws be adopted and become effective as from 1 January 1951."

The resolution was adopted. It was also agreed that the Honorary Officers would nvey to Council the main criticisms of the new by-laws arising from the discussion d that suitable amendments would be formulated by Council for consideration at a further ecial General Meeting to be called in January 1951.

At the second meeting the following resolution from Council was proposed from the hair :

"That consequent upon adoption of the new by-laws the period of office of the present fficers and Council be extended to 30 September 1951."

The resolution was adopted.

Ordinary Meeting, 20 December 1950

The election of the following candidates was confirmed :

Fellows

LEXANDER, Bruce Webster, "Harwin," Grange Road, Fenham, Newcastle-on-Tyne.
NGUS, David Edgar, C.S.I.R.O., Graham Road, Highett, Melbourne, Australia.
IALE, Rodney Barker, 192 Watling Street, St. Albans, Herts.
IARLEY, William Stratford, 315 Bloor Street West, Toronto, Ontario.
MES, David Gwynfor, Ph.D., B.Sc., 2 Glamorgan Street, Barry, Glam.
MES, Peter Arthur Ansdell, 130 Forest Road, Southport, Lancs.
TTERTHWAITE, Edwin, East Mount, 8 Park Road, Ulverston, Lancs.

Foreign Member

Vassy, Professor E. J., 15 Rue Daubenton, Paris, 5e.

Associates

Bonython, Charles Warren, B.Sc., c/o. I.C.I. Alkali (Australia) Pty. Ltd., Solvay Road Osborne, South Australia.
Garnett, F. Lt. Walter Murray, North West Air Command, R.C.A.F., Edmonton Alberta.
Hacking, Glen Cormie, 9920 - 156th Street, Edmonton, Alberta.
Hardman, Maurice John Street, Exeter, Ontario (reinstatement).
Lee, F. O. Eric F. 20503, 435 (T) Squadron, R.C.A.F., Edmonton, Alberta.
McLeod, Owen Kenneth, A.F.O., R.C.A.F., St. Hubert, P.Q.
Taylor, Reginald John, Section of Meteorological Physics, C.S.I.R.O., Graham Road Highett, S.21, Victoria, Australia.
Thomas, Miss Roma Irene, c/o C.S.I.R.O., Section of Meteorological Physics, Graham Road, Highett S.21, Victoria, Australia.

Student Associates

Lake, David Brian, 10-12 Herne Hill, London, S.E.24.
Moore, David James, 31 Wandsworth Bridge Road, London, S.W.6.
Pike, Miss Noreen, 14a Church Road, Wimbledon, London, S.W.19.
Procter, Miss Dorothy Mary, Rye Hill, Ulceby, Nr. Grimsby, Lincs.
Shaw, Derek, 27 Madrid Road, Barnes, London, S.W.13.

The following papers were read and discussed :

(a) The application of the similarity theory of turbulence to atmospheric diffusion by G. K. Batchelor (*Q.J.*, **76**, p. 103).

(b) Advective temperature change in the 1,000-700 mb and 700-500 mb layers by J. M. Craddock (*Q.J.*, **77**, p. 51).

Vacation Meeting, 3 *January* 1951

A vacation meeting was held, intended primarily for Student Members. A talk o "Clouds" was given by F. H. Ludlam and this was followed by a Brains Trust.

Special General Meeting, 17 *January* 1951

A Special General Meeting was held to consider the following amendments propose by Council to the new by-laws already adopted at the Special General Meeting on 1 December 1950 :

1. Delete from by-law 36, lines 7 and 8, the words : "after 30 years' continuou Fellowship."

2. Add at the end of by-law 71 the words : " which shall be held not later than six weeks after receipt of the requisition."

3. Add at the end of by-law 72 the words : " which shall be held not later than six weeks after receipt of the requisition."

The amendments were adopted.

Ordinary Meeting, 17 January 1951

The election of the following candidates was confirmed under the old by-laws :

FELLOWS

UTTLING, GEOFFREY, 8 The Highway, Hawarden, Nr. Chester.
HANDLER, Plt. Off., TONY JOHN, B.Sc., Officers' Mess, R.A.F., Cranwell, Nr. Sleaford, Lincs.
HAPMAN, ALFRED ERNEST, M.Sc., 2 Beresford Road, London, N.8.
ALCONER, THOMAS THOMSON, B.Sc., Dinsburn, Selkirk, Scotland.
AST, Instr. Cdr. RUPERT DONALD, R.N., " Rosedean," Horndean, Hants.
ENSON, FREDERICK ARTHUR, Ph.D., 11 Brookhill Drive, Wollaton, Nottingham.
ONES, WILLIAM TREVOR, 5 Herbert Street, Bridgend, Glamorgan.
EWIS, RODERICK PYRS WALDO, Briarleigh, Longacre Road, Carmarthen.
ICCORMACK, JOHN G., B.Sc., Institute of Science & Technology, Kevin Street, Dublin.
IDDRIE, DAVID LAWRENCE, 41 Woodhouse Road, Pietermaritzburg, Natal, South Africa.
ICHARDSON, WILLIAM ERIC, 16 Foster Road, Thorne, Nr. Doncaster, Yorks.
OBERTS, GLYN ARFON, M.R.I.P.H.H., A.R.San.I., " Rockfield," 4 Howard Rd., Llandudno, Caerns.
INCE, JOHN WILLIAM HENRY, 42 Sharratt Street, London, S.E.15.
ALSHAW, C. DESMOND, M.A., Clare College, Cambridge.
EIGHTMAN, JOHN LEE, 95 Netherby Road, Edinburgh, 5.

FOREIGN MEMBERS

LAMLY, Abdel KADER M., Dept. of Meteorology, Imperial College, S.W. 7.
EUMANN, JEHUDA, M.Sc., Meteorological Service, Hakirya, Israel.
MAR, MAHMOUD HASSAN MOHAMED, Dept. of Meteorology, Imperial College, S.W.7.

STUDENT ASSOCIATES

DAMS, JOHN MICHAEL, " Burbank." Seaford Road, Newhaven, Sussex.
REEN, JOHN WILLIAM, 56 Gloucester Road, New Barnet, Herts.
OWELL, DAVID, B.Sc., Selkirk Hall, 50 Holland Park, London, W.11.

The following candidates were then declared to have been elected by Council in accordance with the new by-laws at their meeting held that afternoon before the Ordinary meeting :

Fellows

Ashley, Flt. Lt. Kenneth Merza, R.A.F. Flying College, Manby, Nr. Louth, Lincs.
Bannister, Richard Courtney, B.Sc., 10 Silveracre, Whitechurch Road, Rathfarnham, Dublin.
Chuter, Ian Henry, 64 West Way, Shirley, Croydon, Surrey.
Day, Geoffrey James, 4 Listley Street, Bridgnorth, Salop.
Griffiths, John Frederick, M.Sc., 29 Ingleton Avenue, Welling, Kent.
Jacob, William Minas, c/o Grindlay's Bank Ltd., 6 Church Lane, Calcutta, India.
Pringle, Mrs. Anne Collier, 40 Learmouth Crescent, Edinburgh.
Ward, Cyril G., 6 St. Peter's Place, Drogheda, Co. Louth, Ireland.
Williams, Tudor, " Goodamoor," Sparkwell, Nr. Plymouth, Devon.

Foreign Members

Arnason, Geirmundur, Vindragarvagen 4 n.b. Stockholm Sv. Sweden.
Rotaba, Mahmoud Hassan, c/o Navigation Section, Civil Aviation Dept., Kasr El Nil, Cairo, Egypt.
Starr, Victor Paul, Massachusetts Institute of Technology, Cambridge 39, Mass., U.S.A.

Student Member

McLennan, Alexander Watt, 27 Restalrig Square, Edinburgh 7.

A discussion was held on " Dynamical methods in synoptic meteorology." The openers were Dr. R. C. Sutcliffe, F. H. Bushby, and E. J. Sumner.

Joint Meeting with Institution of Electrical Engineers, 30 January 1951

A joint meeting with the Radio and Measurements sections of the Institution of Electrical Engineers was held at the Institution of Electrical Engineers, Savoy Place, London, W.C.2. The meeting took the form of a symposium on " Electrical meteorological instruments."

The following papers were read and discussed :

G. E. W. Hartley	The development of electrical anemometers.
H. H. Rosenbrock and J. R. Tagg	Wind and gust measuring instruments developed for wind-power survey.
H. D. Hawkes and J. M. Laws	An electrical method of resolving true wind velocity aboard ship.
J. Bell and E. M. Langham	Mean-wind-velocity instruments for use on naval craft.
F. E. Jones, J. E. N. Hooper and N. L. Alder	The radar-sonde system for the measurement of upper wind and air data.
A. W. Brewer and R. H. Dobson	An automatic frost-point hygrometer for measurements in the upper air.

A. W. Pratt, B. G. Collins, R. E. Lacy and E. W. Spink — Some meteorological instruments used in building research.

The chairman of the afternoon session was Dr. G. A. V. Sowter, and of the evening session, the President, Sir Robert Watson-Watt, C.B., F.R.S.

Ordinary Meeting, 21 February 1951

The following candidates were declared to have been elected that day by the Council :

Fellows

Bourke, P. M. Austin, M.Sc., Irish Meteorological Service, 14 Dean Swift Road, Glasnevin, Dublin.

De Lisle, Flt. Lt. John Felix, R.N.Z.A.F., c/o Dept. of Meteorology, Imperial College, London, S.W.7.

Dunsmore, Martin, 3a Store Road, Ballygunge, Calcutta, India.

Heyes, Albert, 120 Manor Street, Accrington, Lancs.

Lewis, William Desmond, Meteorological Office, Royal Air Force, West Raynham, Norfolk.

Lupton, Sqn. Ldr. Hugh William, R.C.A.F. Training Command Headquarters, Trenton, Ontario.

Marvin, Daniel Peter, Canadian Pacific Air Lines, Vancouver, A.M.F., B.C.

Morgan, William Arthur, M.Sc., Irish Meteorological Service, High Meadow, Foster Avenue, Dundrum, Co. Dublin.

Murray, William A., 59 Pikeman Road, Glasgow, W.3.

Naqvi, Syed Sibte Nabi, M.Sc., Pakistan Meteorological Service, 46 Jail Road, Lahore, Pakistan.

Quadir, Flt. Lt., M. A., Meteorological Section, Royal Pakistan Air Force, Station Peshawar (N.W.F.P.) Pakistan.

Roberts, Sqn. Ldr. John Gerald, D.F.C., D.F.M., R.A.F., 61 Sharps Lane, Ruislip, Middlesex.

School of Cosmic Physics, Dublin Institute for Advanced Studies, 64/65 Merrion Square, Dublin.

Associates

Barclay, Ernest Lawson, D.A.F.O., P.O. Box 545, Moncton, New Brunswick.

Brewer, Donald Austin, A.F.O., R.C.A.F. Station, Centralia, Ontario.

Campbell, Flt. Lt. John Douglas Wier, Officers' Mess, R.C.A.F. Station, Trenton, Ontario.

Decher, F. O. Robert, R.C.A.F. Station, Trenton, Ontario.

Dempster, Sqn. Ldr. John Henry, R.C.A.F. Station, Trenton, Ontario.

Finley, Sqn. Ldr. Eric Duff, 9 Kerr Crescent, Trenton, Ontario.

Gerger, Herman, Meteorological Office, Ottawa Airport, R.R.2, Billings Bridge, Ontario.

Gigliotti, Thomas Fred, Officers' Mess, R.C.A.F. Station, St. Hubert, Quebec.

Holland, Sgt. Ralph Edmond, D.A.F.O., R.C.A.F. Station, Rockcliffe, Ontario.

Hopkins, Edward Sydney, Officers' Mess, R.C.A.F. Station, Trenton, Ontario.

Jardine, Wg. Cdr. Alexander Myles, Officers' Mess, R.C.A.F. Station, Trenton, Ontario.

Lucy, George Arthur, Meteorological Office, Frobisher Bay, N.W.T., c/o Eastern Arctic Patrol, R.M.S., Ottawa, Ontario.

MacVicar, Alex George, Meteorological Office, D.O.T. Administration Building, Montreal Airport, Dorval, Quebec.

and maintenance of equipment or in human labour, or in both. And the facile theories that large-scale changes of weather can be effected by the application of atomic power raise considerable doubts as to the effect of changes effected in one place on the weather of another place. The author sounds a salutary warning of the risks involved in any effort to produce changes of weather on a large scale.

It will be seen that Dr. Brooks has written a book which covers a field not hitherto attempted in any English book, or, so far as is known to the reviewer, in any book in any other language. It gives a useful summary of climatic factors which should make it valuable as a work of reference, and though some of the earlier portions of the book could be improved, it will be regarded as a useful addition to the literature of our subject.

D.B.

Handwörterbuch der Meteorologie. Edited by Dr. Karl Keil. Frankfurt a.M. (Fritz Knapp), 1950. 8vo. Pp. VII, 604 ; illus. 30 Marks.

Dr. Keil, in his new Glossary of meteorology, interprets both glossary and meteorology in a most comprehensive way. Not only are all the terms, both common and rare, specific to meteorology defined, but also a great many from astronomy, geophysics, geology and other cognate sciences. The 3,500 entries include also brief biographical details of a number of meteorologists from Aristotle onwards. These are limited to dates and places of birth and (if not still living) death ; some might consider them out of place in a glossary but others would have liked to see them extended by brief references to the meteorological work for which the subject is famous. Incidentally, it was news to the reviewer that Tor Bergeron was born at Godstone and so may be regarded in some sense as a British meteorologist. There are also some useful histories of meteorological institutions and periodicals. The definitions are simply and clearly written ; they are described as intended for scholars, farmers, *etc.* interested in meteorology through listening to broadcasts, but most professional meteorologists should learn a good deal from the volume. The array of terms, which many students find bewildering, for the thermal state of the atmosphere and its " temperature "—equivalent, potential *etc.*—are elucidated fairly well, but some will be puzzled by the recondite distiction between " potential-equivalent temperature " and " potential equivalent temperature " and it might have been explained that the distinction is, practically speaking, only a matter of wording. The book contains many other examples of the continual coining of new words by meteorologists, which is rapidly making meteorology almost as unintelligible as ancient Icelandic to the general scientist. There are also innumerable names for local winds and other phenomena ; one which was new to the reviewer is *asgardsweg* for the destructive path of a tornado.

There is very little to criticise. The Royal Meteorological Society's *Bibliography of Meteorological Literature* is ascribed solely to the Meteorological Office, and brought to a premature end in 1936. " Pea Soup " fogs are surely British and not American in origin. There is a slight error in Thornthwaite's formula for rainfall effectiveness. But these are trivial ; essentially the Glossary appears to be very sound, and an immense amount of research must have gone to its compilation. Some of the items, *e.g.* actinometer, include interesting historical notes, and quite a number end with a reference for further reading. All meteorologists who read German should try to obtain a copy.

C.E.P.B.

The Thunderstorm. Report of the Thunderstorm Project (a joint project of four U.S. Government Agencies : Air Force, Navy, National Advisory Committee for Aeronautics, and Weather Bureau). Washington, D.C. (U.S. Government Printing Office), June 1949. 8vo. Pp. IX, 287 ; 254 figures, 49 tables. $2.25.

This book is the final report on the present phase of the ambitious scheme of research known as the *Thunderstorm Project*. The project is intended primarily to get information for the benefit of aviators on the internal structure and the behaviour of thunderstorms, but the present report is of more general interest, and provides a detailed description, based on new measurements, of many of the physical processes involved in the most active form of atmospheric convection. One summer was spent investigating thunderstorms over an area in Florida where air-mass thunderstorms are common, and a second summer in Ohio, where frontal thunderstorms occur. In all, 179 storms were investigated the instruments of investigation including a network of radar and upper-air sounding stations, a much denser network of surface observing stations with autographic instruments and a flight of aircraft equipped with special instruments with which repeated traverses were made horizontally through any chosen thundercloud at 5,000 ft intervals up to 25,000 ft above ground level. The collection and

reduction of this data required a large organisation, and at least 80 persons are mentioned by name for their shares in it. The present report however is mainly the work of Roscoe R. Braham, Jr. and three collaborators, and shows a commendable unity of style and outlook.

The report is in three parts, dealing respectively with the physics of the thunderstorm, the hazard presented to aviators and with a number of particular storms, each analysed in very great detail. An appendix gives particulars of all the instruments used. The first section includes a description of the dynamical and thermal structure of active convective cloud, the associated surface weather and and electric field and the effect of the environment on the formation and growth of thunderstorms. Some of the conclusions are new, but most are results already suggested by physical reasoning which now are established by adequate observations. The present reviewer attaches most importance first to the demonstration, beyond reasonable doubt, that the thunderstorm consists of a group of cells at different stages of development, each having dimensions of a few miles at most, and passing through a characteristic life cycle, and secondly to the provision of an adequate physical explanation for the development within the cell of a downdraught in association with the heaviest precipitation. This downdraught, on striking the ground, spreads out as the cold gust which the surface observer recognizes as a forerunner of the storm, while the development within the cell of the downdraught at the expense of the updraught characteristic of the developing cumulus eventually brings the active life of the cell to an end.

The second section, which is of direct interest to pilots as well as meteorologists, leads to the general conclusion that a competent pilot in a sound aircraft can traverse a thunderstorm in safety, if in discomfort, by following some fairly simple rules.

Looked at as a whole, the report is notably successful in presenting in some order an enormous mass of observational material, and deserves consideration by the serious student of any of the many atmospheric processes with which it is concerned. The presentation including printing and binding is excellent.

J.M.C.

Reports on progress in Physics, Vol. XIII. London (The Physical Society), 1950. 8vo. Pp. 424 ; illus. 50*s*.

The last number of this invaluable annual contains several articles of interest to the student of atmospheric physics.

L. Goldberg describes recent advances in solar infra-red spectroscopy, where a new field has been opened by the improvements made in the past few years in the sensitivity of infra-red detectors. The article reviews the whole field somewhat cursorily and describes, in detail, the results of one particular group of investigators. While the result is scarcely a balanced survey, it serves to bring home vividly the dramatic advances which have been made ; in particular, use of the lead-sulphide cell makes it possible, in the region of wave-lengths out to about $3{\cdot}5\mu$, to approach the theoretical resolving power of a grating spectrograph.

Recent experimental investigations have confirmed the theoretical prediction that, as one proceeds to increasing wave lengths, the number of atomic solar lines of appreciable intensity falls off rapidly beyond about $2\,\mu$, and the complicated spectrum observed thus becomes, practically, a band spectrum of the terrestrial atmosphere. Much of its complex structure has already been elucidated as due to familiar polyatomic atmospheric gases together with their modifications resulting from the replacement of one of the atoms by a rarer isotope. Thus heavy water, HDO, and carbon dioxide, with a ^{13}C or ^{18}O atom, are the causes of major features of the atmospheric spectrum. In addition, nitrous oxide and methane have emerged as permanent constituents of our atmosphere, in appreciable quantity, although there is no evidence that they, like ozone, are concentrated in the upper atmosphere.

G. P. Kuiper summarizes our knowledge of planetary atmospheres in a fascinating article illustrated by beautiful photographs. The earth is specifically excluded but the article contains interesting discussions of planetary meteorology and climatology and the wealth of reliable information now available will probably astonish any who are not in touch with modern developments in this field.

Mars, to choose a planet at random, owing to the considerable eccentricity of its orbit, shows marked differences in climate between its two hemispheres, the southern one having a short warm summer and a long cold winter. Spectroscopic evidence indicates the presence of carbon dioxide in its atmosphere and shows that the famous polar caps do indeed consist of deposits, probably quite thin, of frost at a very low temperature. The Martian day is about the same length as ours, the year about twice as long as our own. Atmospheric pressure near the ground is about 10 mb, the frost point about —50°C, and the total precipitable water in the atmosphere less than 0·05 mm. Maximum temperature, in the equatorial regions, is about 0°C and the minimum about —70°C. Despite the low water-vapour content of the atmosphere, and the absence of detectable areas of liquid water, the

planet is normally veiled by a bluish-white haze, probably of ice crystals, which occasionally clears locally. Clouds are relatively rare but two types have been recognized ; high whitish clouds, apparently an intensification of the general haze layer, ordinarily appear, at heights between 4 and 20 km, shortly before sunset and disappear shortly after sunrise. It is anticipated that polarisation studies will soon yield evidence on the crystal shape of the cloud particles ! Occasionally, large areas are covered by a different type of low-lying yellowish cloud, perhaps consisting of dust particles. Cloud motions indicate winds of the order of 20 mi hr^{-1}.

When one of the polar caps is evaporating rapidly, in spring, the adjacent dark areas show a well defined green coloration. It is plausible to ascribe this to vegetation, since carbon dioxide and water vapour are now known to be present in the Martian atmosphere. Spectroscopic studies rule out the possibility of the coloration being due to vascular plants of the terrestrial type but would permit something of the nature of lichens or mosses.

C. F. Powell in an article on mesons (unstable particles with a mass intermediate between that of a proton and an electron), gives the latest news of the search for new fundamental particles, a field in which he has made a major contribution. Mesons can be claimed as an atmospheric phenomenon, having been discovered in the cosmic radiation. There is now strong evidence for the existence of mesons of three different masses, and with lifetimes varying from 2 microsec to something less than 5×10^{-14} sec. All three types can carry positive or negative charges and at least one of them can exist as a neutral particle !

T.W.W.

Le rythme des climats dans l'histoire de la terre et de l'humanité. By Ed. le Danois. Paris (Payot), 1950. 8vo. Pp. 204, 18 figures and maps. 480 fr.

Climate in geology and history is the theme and rhythm the keynote of this readable book. Ice Ages occur every 200 million years because (according to Couderc) the sun rotates around the Milky Way in that period. Pettersson's (1914) tidal theory provides the favourite cycles, and the effects of varying positions of the sun, moon and earth are set out in charts and tables.

The tidal curve, like that of Milankowitch, offers a time scale of climatic change. Deteriorations occur at A.D. 1430, 425 B.C., 2265 B.C., which three instances, as Brooks pointed out, roughly fit the facts. In the manner of Huntington, Dr. le Danois sees the same curve in fashions and in prosperity. He notes *inter alia* light clothes in the (warm) twelfth century ; (warm) luxury replaced by (cool) severity in the mid-sixteenth ; short skirts in the (warm) 1920s. Unfortunately, he fails to quote authorities and his tidal curve often conflicts with climatic facts.

The value of the book lies not in its theories but in the stimulating ideas on a variety of subjects (*e.g.* D-day tides, p. 31 ; the oyster failure of 1921, p. 71) by a well informed oceanographer.

D.J.S.

Methods in Climatology. By V. Conrad and L. W. Pollak. 2nd Ed. Harvard University Press, 1950. 8vo. Pp. 459 ; 115 figures, 101 tables. $7·50 (48s.).

We are told in a recent book* that the activities of the working week are governed by "climate," those of weekends and holidays by "weather." This jaunty contrast, emphasising as it is meant to do the long-term character of climate, even so serves as a reminder that climate is composed of variations.

Long ago, the all-important variation was the seasonal one. Biological studies† recount how the musk ox of North America, for instance, finding feeding grounds overcrowded, moved northwards in spring to enjoy the delicacies of fresh herbage but newly uncovered by the seasonal retreat of winter's snow. The nomadic hunter in migrating after him, experienced a seasonal variation of locality in step with that of climate. Later, human habitat seems to have been determined by average annual conditions. The reviewer read recently‡ that up to mediaeval times, the main centres of civilisation and culture were more often than not restricted to regions intersected by the annual isotherm of 70°F. But, as more efficient methods of indoor heating evolved and the optimum climate was discovered further and further poleward, seasonal variation again came into prominence. It influenced now, not so much man's place of habitation as his architecture and his activities. It is in the stormy zones of our own latitudes that civilization has reached its highest development and man is most active both mentally and physically ; he has never been unaffected by climate ; in recent times, however, he shows himself largely undaunted by it.

* Brooks, C. E. P., 1950, *Climate in everyday life* (Benn).
† *e.g.* Lockley, R. M., 1947, *Letters from Skokholm* (Dent).
‡ Markham, S. F., 1947, *Climate and the energy of nations* (O.U.P.).

The very overcoming of climate makes civilization increasingly dependent upon climatology. Maintaining a healthy population and providing it with amenities and luxuries at reasonable cost demands intensive study of a wide range of climatic factors. A knowledge merely of average seasonal variation is inadequate for present-day concerns. Not only are extreme values of interest, serving as a guide to constructional engineers ; but, for numerous pursuits and products, some idea of general variability is needed ; and this, not for individual climatic elements only — temperature, rainfall, wind speed, cloudiness and so on — but also for two or more elements conjointly. Examples of variations studied in individual elements are local distribution of fog, a vital factor in the siting of an airfield ; and frequencies of wind direction, required for the planning of runways. As an example of the study of combined elements, decisions as to the quality of packing to be used in specific regions for films or rubber balloons may be cited. For such decisions, associated values of temperature and humidity are required. Again these two elements combined with wind speed determine " drying power " and the effect of climate on human energy.

There are many excellent published works on climatology and climate — Buchan, Braak, Hann, Köppen und Geiger, Shaw, Ward, Bilham, Brooks, Kendrew are " household words " among meteorologists — but these works are insufficient to meet all the demands made upon climatologists. In the first place, textbooks on climate grow out-of-date as more observations become available. During the past decade, stations reporting " surface " meteorological observations have increased appreciably in number and those reporting upper-air observations even more so, in proportion. Fifteen years ago the Köppen-Geiger *Handbuch der Klimatologie* and Vol. II of the *Manual of Meteorology* gave information about the upper air that was then amazingly comprehensive ; it looks puny now when compared with the mass of upper-air data accumulated all over the world in the past six to eight years. There will always be a demand for new summaries of climatic data and the inexperienced compiler needs guidance in the basic methods for the computing and graphing required in such " climatographies." Secondly, a textbook cannot be expected to solve all the climatological problems that arise in aviation, agriculture, commerce, industry, medicine and other fields in which weather effects may be of importance. Each potential climatologist (be he meteorologist, geographer or hydrologist) must learn for himself to select, interpret, and occasionally, extrapolate meteorological data. A handbook of basic methods might usefully give instruction in all these things. Such needs are to a large extent met by Conrad and Pollak's *Methods in Climatology*.

The work of summarizing, interpreting and extrapolating involves at least three separate processes : first, the critical examination of observational material ; secondly, recognition of essential details and representation of results ; and thirdly, estimation. The first requires information concerning methods of observing ; for example, differences in site, type or condition of an instrument may affect appreciably the readings recorded. For the second and third, there is a choice of technique to be made, based largely upon the work of previous investigators ; particular requirements are, on the one hand, appreciation of the adroit use of tables, graphs and diagrams and, on the other, familiarity with the parameters and with some of the applications of mathematical statistics.

Brief notes on the organisation of a world-wide network of stations and on meteorological instruments and observing are given in the opening chapter of *Methods of Climatology*, an interesting feature of which is the comparison of raingauges in use in different countries.

Graphical methods of presenting data, accompanied by references to original papers showing applications of the methods, are included in chapters dealing with selected elements and spatial comparison. No hints are given on the compiling of tables ; but the ones appearing in the text, like most of the diagrams, are clear and easily understood.

Statistical concepts, such as frequency curves and the parameters associated with them, are introduced early in the book ; and, of statistical methods, curve fitting, harmonic analysis and correlation are given in later chapters. Thus, at a conveniently simple level, the book is seen to answer many of the demands of practical climatology.

It must not be supposed that professional meteorologists have been unaware of the need for a text on methods ; had they been, the reviewer might be accused of a false justification of the book under review. Two editions (1917 and 1931) were published of D. Brunt's *The combination of observations*. An interesting account of the use of cartesian co-ordinates, nomograms, isograms, harmonic analysis and correlation coefficients is given in Vol. I of Shaw's Manual ; and an extensive bibliography on graphical methods in meteorology is shortly to appear in *Meteorological Abstracts and Bibliography**. Among collections of statistical methods appearing in official periodicals, one may mention an article as early as 1916 by C. F. Marvin† and a recent series by C. E. P. Brooks‡. None

* Published by the American Meteorological Society.

† Elementary notes on least squares, theory of statistics and correlation for meteorologists and agriculturists, 1916, *Washington Mon. Weath. Rev.* **44**, p. 551.

‡ The numerical basis of climate, 1947, *Met. Mag.* **76**, pp. 9, 36, 56.

of these, however, gives the same range of methods applied exclusively to climatology as does the publication now considered, which, as regards subject matter, lies between two textbooks now being planned by the London Meteorological Office to replace the methodological parts of the old *Computer's Handbook*, a textbook of elementary computing on the one hand, and of statistical methods in meteorology on the other. The latter, already in the press, includes the commonly-used significance tests, analysis of variance and an examination of " persistence " in meteorological phenomena, topics omitted even in the revised edition of Conrad and Pollak.

Such omissions are not, however, to be considered a failing of *Methods in Climatology*. Apart from the chapters which have been added on " periodography," the new edition retains the simpleness and directness of the former edition ; and a volume which includes a very elementary discussion of the drawing of isograms has not room for more than a few of the simpler products of mathematical statistics, if it is to be kept both consistent and within reasonable limits as to size.

The presentation of Part I of the present edition reflects the aim, stated by Conrad in the first edition, of offering " a bridge connecting the two realms " of geography and climatology. The main theme, culminating in Chapter X — The essential elements of the climatography — is that of providing climatological tools for the geographer. This restricted outlook belies the scope of the book. It would be more alive if some of the many other applications of climatology also were stated. As it is, original papers and articles have to be looked up before the reader grasps the full purpose of one or two of the methods outlined.

The second edition contains a useful new chapter on graphical methods as aids to computing and another on mechanical and electrical devices. The reviewer regrets the inclusion of lengthy chapters on hidden periodicities, adding to the cost of the edition ; but the only irksome features of the book as a whole are a lack of care in nomenclature and choice of symbols and an over-frequent use of footnotes. The paper is of improved quality, the welcome abundance of bibliographical references has been balanced by the inclusion of a few more British authors and the numerous diagrams are clear and interesting. The volume will be a welcome addition to the bookshelf of every meteorologist — provided that he can spare 48 shillings.

N.C.

Recent additions to the Society's Library

British rainfall 1943, 1944, 1945. Meteorological Office, M.O. 505. London, His Majesty's Stationery Office, 1950. Pp. v, 278, maps, tables.

Co-operative snow investigations, Progress report 1945—1950. Oakland Army Base, California, 1 June 1950. Pp. VI, 64, illus., tables.

RUDOLF GEIGER, *Das Klima der Bodennahen Luftschicht*, 3rd edition. Braunschweig, Vieweg, 1950. Pp. VIII, 460, 195 figs., tables.

London essays in Geography. Edited by L. Dudley Stamp and S. W. Wooldridge. London, Longmans Green, 1951. Pp. XIV, 351, figs.

Procès-Verbaux des Séances de l'Association de Météorologie ; I. Actes de l'Association, II. Mémoires et discussions. Union Géodésique et Géophysique Internationale, huitième Assemblée Générale, Oslo, Août 1948. Uccle, Belgium, A.I.M. No. 8c, 1950. Pp. 188, figs.

Reports on progress in Physics, Vol. XIII. London, The Physical Society, 1950. Pp. 424, illus.

LAURI A. VUORELA, *Synoptic aspects of tropical regions of the Atlantic Ocean, W. Africa and S. America*, The Aerological Atlantic Expedition of the Finnish Academy of Sciences Helsinki, University of Helsinki. Institute of Meteorology, 1950. Pp. 130, IX, illus., 14 tables.

Obituary

Mr. C. DICK PEDDIE, C.B.E., M.A., LL.B.

By the death at Edinburgh on 3 May 1950, in his 87th year, of Coventry Dick Peddie the Society lost a distinguished senior Fellow. Mr. Dick Peddie, fourth son of the well-known architect John Dick Peddie, R.S.A., joined the Scottish Meteorological Society in 1910 and was among those who transferred their membership to the Royal Meteorological Society on the fusion of the two bodies in 1921. Educated at Edinburgh Academy. Cargilfield, Fettes College, and the University of Edinburgh, where he took the M.A. and LL.B. degrees, Dick Peddie was called to the Scottish Bar in 1889. After 11 years' conspicuously successful forensic work he received the appointment of secretary to the Northern Lighthouse Commissioners in 1901. This post he held until March 1929, when he retired. For his long-continued able services he was created C.B.E. in the New Year Honours of 1930. Mr. Dick Peddie's interests ranged over a wide field. He was an all-round sportsman, a journalist of note, an accomplished painter and caricaturist, and, in his spare time, an engineer. For six years he was reporter on the staff of the Scots Law Times.

E.L.H.

Mr. A. HAMPTON BROWN

A good and faithful friend of more than half a century's standing is lost to the Society by the death of Arthur Hampton Brown. He had been in frail health for some time and after rallying from a severe attack of bronchitis during the autumn of 1950 he suffered a relapse and passed peacefully away on the morning of 25 November 1950 at Trimmers Hospital, Farnham. Hampton Brown was born in Chelsea on 15 March 1875. He received his education partly at home and partly at schools nearby. During boyhood and youth his great interest was music, but being by force of circumstances self-taught except for a few lessons in organ playing he felt that his prospects of making a successful career of this hobby were slight. He therefore decided to seek other means of livelihood. Quickness at figures coupled with exceptionally clear and neat handwriting secured him a junior clerical post at the Meteorological Office. This appointment he resigned in 1896 to join the staff of our Society as clerk and computer under the direction of the Assistant Secretary, William Marriott. In those days the Society had its own network of climatological stations distributed over England and Wales. One of Hampton Brown's chief duties was to check the data received from these and to draw up summaries of the observations for the quarterly publication known as the *Meteorological Record*. Mainly in his hands, too, was the preparation of the British entries for the meteorological section of the Royal Society's International Catalogue of Scientific Literature—an enterprise in which our Society and the Meteorological Office collaborated over a long term of years. He also took part-charge of the Society's library and correspondence. All these duties he performed ably and with conscientious care. It thus happened that when Marriott retired from the post of Assistant Secretary in 1915 the Council had to look no farther than Hampton Brown for an efficient successor. Soon after he had taken Marriott's place the man-power demands of the 1914-18 war made their call on his knowledge and experience. He was commissioned lieutenant in the R.N.V.R. for meteorological duties on behalf of the Royal Naval Air Service. Fortunately for the Society his allotted tasks lay at the Admiralty, so that he was able to retain his post as Assistant Secretary and carry on the Society's business in his spare time. These were laborious days for Hampton Brown : often did his dual set of duties keep him at his desk until far into the night. But he was never heard to complain.

The years immediately following the First World War were a notable epoch in the Society's history, witnessing such important events as the transfer of headquarters from Victoria Street to Cromwell Road in 1921, the amalgamation with the Scottish Meteorological Society in the same year, and the 75th anniversary celebrations in 1925. To Hampton Brown fell much of the heavy administrative work involved thereby. All this he undertook to the Council's full satisfaction. His intimate knowledge of the Society's affairs proved of great value to a long succession of presidents, honorary secretaries and honorary treasurers ; tributes to his help and courtesy were repeatedly paid either from the chair at Annual General Meetings or in the Reports of Council. He also made many friends among Fellows who visited the Society's premises in quest of information and sought his aid, which was always willingly given. It was thus a matter for general regret when increasing deafness and a wish to move from London into the country led him to resign his post on 31 December 1935, after 40 years on the office staff. At the Annual General Meeting in January 1936 his retirement on pension was announced, and in token of the Fellows' esteem and appreciation of his long-continued services for the Society he was presented with an inscribed gold signet-ring and a cheque. He was at the same time elected to unsolicited life Fellowship — an honour which gave him particular pleasure.

In 1936 Hampton Brown went to live near Farnham, Surrey, taking with him the Stevenson reen, thermometers and raingauge which he had long used for climatological work at Chelsea. e started records in the garden of his new home and contributed regular returns month by month the Meteorological Office. In his speech of thanks for the presentation made to him on retirement expressed the hope that the Society would still find some use for his services. This hope was nply fulfilled. For over 14 years he volunteered constant assistance with the Society's sub-editorial rk : this help was of immense value during the war, when shortage of staff made the regular oduction of the *Quarterly Journal* a difficult matter. Until within a few days of his death he was gaged in checking the proofs of the *Centenary Proceedings*. Early in 1950, though suffering from or health and seldom able to leave his room, he accepted the heavy task of proof-reading the new lition of Inwards's *Weather Lore*, then about to be issued for the Society.

Despite his early fears, Hampton Brown succeeded in turning his musical abilities to good account. hen still a young man he was appointed organist and choirmaster at the church of St. Simon Zelotes, pper Chelsea. This post he occupied with distinction for 25 years. Anthems and solos of his n composition were often performed at the services and festivals of the church. Unpretentious ough these works were, they made an appeal through their sincerity and the deep religious conviction nderlying them. The music script in his fair copies is of almost unsurpassable beauty and clearness, flecting the spirit of an artist intent upon giving of his best to everything he undertook.

This Society, whose welfare had for so long been Mr. Hampton Brown's close concern, is a siduary legatee under his will, being eventually to benefit from his estate to the extent of £1,000 x-free. At the funeral service held in the parish church of Rowledge, near Farnham, on 9 November the Society was represented by Wing Commander R. M. Poulter, O.B.E., Honorary cretary ; Mr. W. M. Witchell, Honorary Treasurer ; Commander C. R. Burgess, O.B.E., R.N., id Mr. W. E. Livock.

E.L.H.

Mr. H. N. WARREN, O.B.E.

Herbert Norman Warren, Director of the Meteorological Branch of the Australian Public Service nce 1940, died at the age of 62 on 5 August 1950. He fell seriously ill while visiting Lausanne attend a meeting of the Executive Council of the International Meteorological Organisation in May. mewhat benefited by a stay in a Swiss clinic, he came to England towards the close of June and soon iled for Australia ; but his illness gained on him during the homeward voyage, terminating fatally hen the ship reached Adelaide. Warren was educated at the University of Melbourne, his native ty, and as a youth he spent a few years in the Victoria Scottish Regiment. Later he entered the epartment of Home Affairs of Victoria. During the 1914-18 war he served as a company commander ith the Australian Forces. After the conclusion of hostilities he held a number of appointments the Public Service, his most important office being membership on the Board for the Organisation f Australian Commonwealth Departments. In 1938 Warren was attached to the Meteorological ranch for duties concerned with its reconstitution for aviation and defence ; in 1939 he became ssistant Director (Administration), and in 1940, as already stated, he proceeded to the directorship. uring the war of 1939-45 he was at the head of the Allied Meteorological Services in the south-west acific ; in 1943 he was made chairman of the Meteorological Committee set up at G. H. Q. there, le held the rank of Group Captain in the Royal Australian Air Force and received the award of the .B.E. for his valuable work.

After the war Warren gave generously of his time and energies to the furtherance of international o-operation in meteorology, travelling many thousands of miles to take part in conferences. At Vashington in 1947 he was chairman of the committee charged with the responsibility of drafting he Convention for the new World Meteorological Organisation. Paying tribute to his memory in a ecent issue of the *Meteorological Magazine*, Sir Nelson Johnson wrote : " Those who had the good rtune to know Mr. Warren recognized in him a man of strong character and robust common sense. His independence of thought made him the champion of any cause which he regarded as worthy f support, and his sense of fairness earned for him the respect of all." Mr. Warren was elected Fellow of this Society on 19 June 1946.

E.L.H.

Mr. W. SEDGWICK, M.A.

Walter Sedgwick, who died on 29 December 1950 after an illness of three years' duration, had eld Fellowship in this Society since 20 January 1909 and served as one of its honorary secretaries om 1916 to 1918. Born in 1877 and educated at Bilton Grange, Uppingham, and Pembroke College,

Cambridge, he passed the remainder of his working life as a parliamentary agent ; for a long period he was chief partner in the firm of Rees and Freres. Mountaineering occupied the bulk of his spare time, and he was an enthusiastic member of the Alpine Club. His main meteorological interests were concerned partly with this activity and partly with the historical aspect of climatology. He contributed to Vol. 46 of *Symons's Meteorological Magazine* a useful series of articles on English weather in the 17th century as recorded by contemporary diarists, initiating a discussion on climatic change which may still be consulted with profit. On Sedgwick's resignation from honorary secretaryship of this Society the president, Dr. H. R. Mill, paid him the following tribute at the Annual General Meeting in January 1918 : " The services of the retiring secretary, Mr. Sedgwick, had been particularly valuable on account of his experience in matters of organisation and of his admirable qualities in getting work done in an easy and effective manner."

E.L.H.

Captain H. A. STAPLES, M.C.

Captain Henry Arthur Staples, a Fellow of the Society since 17 January 1923, died suddenly on 7 November 1950. Trained as a mining engineer, he became a colliery manager in South Wales and later was appointed mineral valuer at the Bristol office of the Inland Valuation Department. On the outbreak of war in 1914 he entered the army and went to France, soon receiving a commission in the London Irish Rifles. In 1916 he was awarded the Military Cross under the following citation : " On several occasions he had personally conducted ammunition and bomb-carrying parties to the front line under intense fire, and had previously done very fine work." After the cessation of hostilities Captain Staples gave much of his time to meteorology and cognate subjects, developing theories of his own which were published in a book entitled *Cosmic machinery in an electro-magnetic universe* (Clowes, 1939). Despite severe criticism these studies continued to engross his attention, and until overtaken by fatal illness he persisted in the endeavour to support his unorthodox findings by proofs that would bring conviction to opponents.

E.L.H.

Mr. C. J. P. CAVE, M.A., F.R.A.S., F.S.A., F.R.P.S.

Charles John Philip Cave, who died on 8 December 1950, was born in 1871, educated at the Oratory School, Edgbaston and at Trinity College, Cambridge. Elected a Fellow of the Society in 1899, he first became a member of Council in 1909, and was President in 1913-15 and again in 1924-26. He married in 1895 Wilhelmina Kerr, who was one of the very few women to serve on the Council of the Society. There were four sons and a daughter of the marriage. Mrs. Cave died in 1944.

Cave did great service to meteorology, service which is quite inadequately represented by his published works, despite the fact that of the 35 volumes of the *Quarterly Journal* published between 1906 and 1940 no less than 26 contain at least one substantial contribution from his hand. These papers fall into three scientific subject groups : upper-air research, thunderstorm research and cloud studies, with a fourth group on organisational aspects of meteorology.

His first paper, jointly with W. H. Dines in 1906, discussed nine kite ascents by which he had made upper-air measurements in Barbados in April and May 1904. Pilot-balloon and registering-balloon ascents, in Barbados and in the United Kingdom, were discussed in papers of 1909-12 ; a very useful account, for the guidance of other workers, of how he carried out two-theodolite ascents, was published in 1910 ; while his first Presidential Address, in 1914, was on " Upper-air research." In 1919, with J. S. Dines, he contributed " Further measurements on the rate of ascent of pilot balloons", describing measurements made in the Albert Hall.

The first of his thunderstorm papers appeared in 1912 ; a 1923 paper, jointly with R. A. Watson-Watt, on " The study of radiotelegraphic atmospherics in relation to meteorology " was the first paper to be published on the location of thunderstorms by radio direction-finding. Another 1923 paper, " Winter thunderstorms in the British Isles " was the foundation paper for the Society's work on winter thunderstorms and the precursor of the thunderstorm census work still in progress in the devoted hands of S. Morris Bower.

His 1917 paper on " The form of clouds," two notes of 1919 and 1922 on particular cloud phenomena, and his pictorial Presidential Address of 1926, when as the Editor records, " Mr. Cave then showed a large and interesting collection of slides," were evidence of that rare combination of profound knowledge of the form and formation of clouds, high technical skill in photography and a keen sense of beauty, which made him the most distinguished cloud photographer of an era when there were

t least two other cloud photographers of high distinction in Great Britain. And, as in the case of pper-air research, he set down briefly and clearly for the guidance of others the methods which he imself used with such outstanding aesthetic and scientific success.

The fourth group of papers began with a 1920 paper on "The status of a Meteorological Office and its relation to the State and the public," continued in 1922 with "Some notes on meteorology n war-time," "The present state of meteorology and of meteorological knowledge" — the 1925 Presidential Address in the seventy-fifth anniversary year of the Society — and the brief remarks n "The activities of the Royal Meteorological Society," which preceded the display of cloud photoraphs which was the main part of his Presidential Address of 1926. All four of these papers merit ery careful re-reading, for the problems of giving the State a meteorological service "as good as modern cience and enterprise might make it," of the relation of such a service to its many consumers in many Departments of State with very diverse interests, of the relation of the scientific civil servant to the dministrative and executive civil servants and to Ministers, to the fighting man in the field and to the armer in the fields, remain unsolved, or at best uneasily compromised. The reader of 1951 will iscern the shadow of Airmet over those pages of 1920-26. No one in the State meteorological ervices (for they are plural) can fail to appreciate the continuing dangers to which C. J. P. Cave drew ttention. No member of the public can fail to be grateful for the wisdom that has kept them in check nd given us a Meteorological Office whose worth we most gratefully recognize, and to which we pay he most appropriate of compliments when we insist, with C. J. P. Cave, that we have the right, the uty and the power to make it still better.

This already catholic series of contributions is most pleasantly rounded off by two other communiations to the Society, "Quotations from the diary of Samuel Pepys on the weather" and "Weather otes in Swift's 'Journal to Stella'."

Cave made a notable pioneer contribution to meteorology in his book *The structure of the atmosphere in clear weather* (C.U.P., 1912) which presented results from two hundred pilot-balloon ascents rom Ditcham Park on the South Downs. He found five main types of structure: (a) "solid current" bove the first kilometre, (b) continual increase in velocity, rising far in excess of the gradient wind, c) decrease of velocity in the higher layers, (d) reversals or great changes in direction at high levels, e) upper winds blowing outward from the centres of areas appearing on the surface chart as low-pressure reas. His statement that the strongest winds are found just below the stratosphere is perhaps the irst foreshadowing of recent observations on the now perhaps somewhat over-dramatised "jetstream."

This summary of Cave's meteorological publications is fittingly completed by a reference to his *Clouds and weather phenomena, for artists and other lovers of nature* (C.U.P., 1926 and recently reprinted), containing not only a beautiful and satisfying selection of his own cloud photographs, but lso friendly and salutary advice to artists on the avoidance of meteorological (and astronomical) olecisms.

But, as was indicated at the beginning of this summary, the written word falls far short of displaying is uniquely personal contributions to meteorology. When his old teacher and friend Napier Shaw rote in the public press calling for the cooperation of yachtsmen and of such lovers of nature as had the eisure, means, facilities and tastes appropriate to the advancement of meteorology by flying kites to ound the upper air, Cave was alone in replying, but he did work that might not have been achieved y a dozen volunteers of poorer metal.

And when he accepted a modest rank in the Meteorological Section of the Royal Engineers in the ar of 1914-18, he did something that could only have been achieved by someone with his background nd his irresistably charming and lovable personality. He acted as a kind of interpreter between the lain honest soldier, with a vague conviction that he needed some help about weather lore, and the rofessional meteorologist, with a clear conviction that he could give much help by weather science, eparated as these two were by the lack of a mental language in common. Only a transparently honest, isarmingly modest, quietly but passionately enthusiastic "middleman" familiar with both these orlds and some others could have ensured the success of this early experiment with civilians in niform. The same qualities made him a gifted and much-loved instructor of the young meteorogist, in days when schools of meteorology were visible almost exclusively through Dickensian 'winders;" meteorologists spelled as they cleaned as they spelled.

Cave did not rest on the laurels of a uniquely distinguished amateur meteorologist of professional ompetence. He conquered another world, and established a reputation which might alone have atisfied a man far less modest than himself, when he turned his sure taste, his telephoto lens and his lectric spotlight on the roof bosses of British and European ecclesiastical buildings, bringing to the wentieth-century reader of his lovely and fascinating book on *Roof bosses in mediaeval churches* (C.U.P., 948) an intimacy with their beauty and their symbolism that could come only from a better view of hem than had been possible for any but the mediaeval craftsmen who fashioned and installed them.

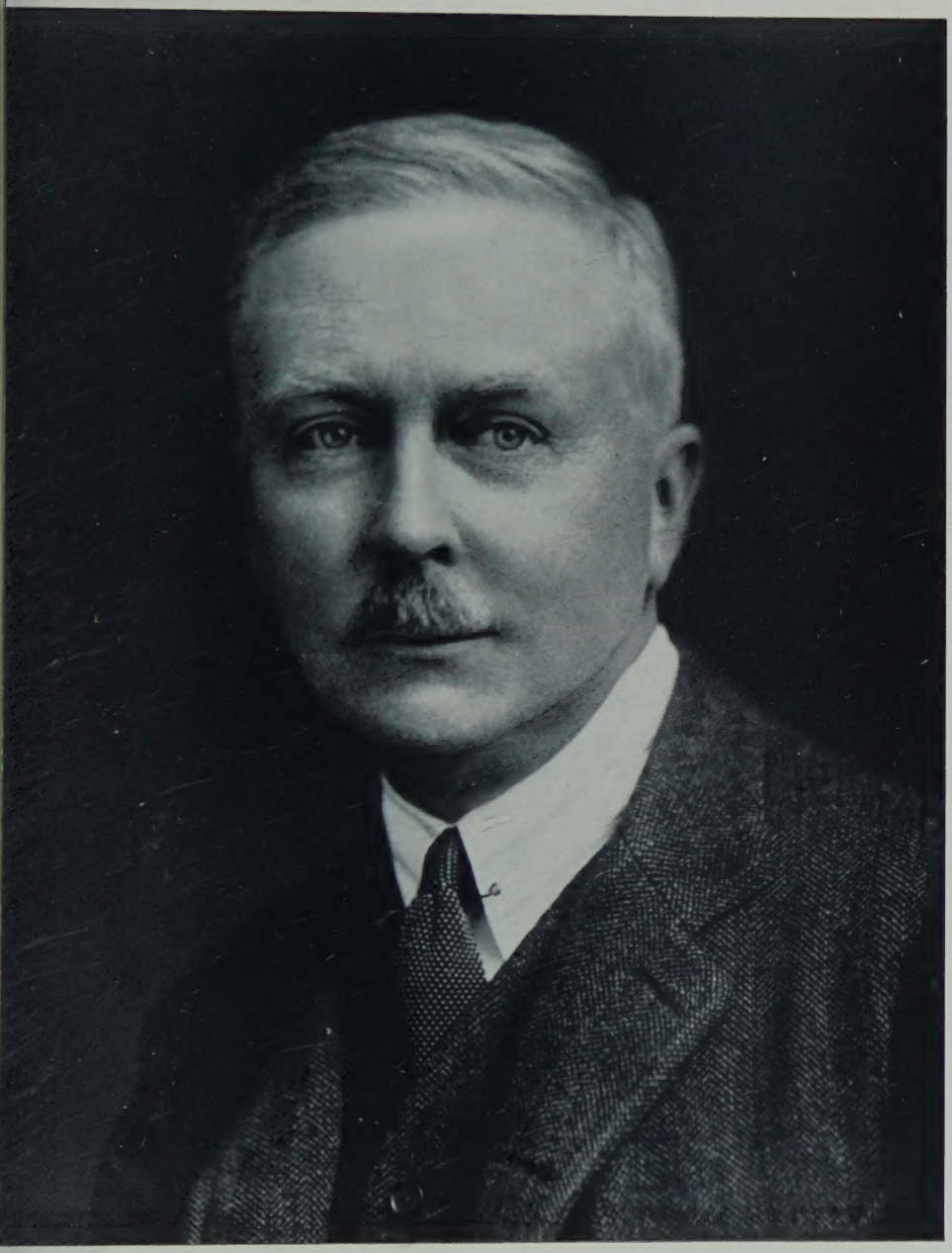
Cave's life has a lesson, above all, for those unimaginative pundits who now address us on "in-

centives " which they view through the wrong end of a sociological telescope. A " gentleman of leisure," comfortably endowed with possessions, he was an amateur, a dilettante, putting his hand to many and divers things that did not obviously enhance " productivity " in its baser sense. What " incentive " had he to invest his material wealth in the diffusion of intellectual and aesthetic wealth among all who cared to share it ? Only, one supposes, the firm conviction that the enrichment of the mind of man everywhere was the route to an immaterial " productivity " without which economic productivity was dust and ashes — and ashes without fire.

He was a gentleman of leisure who toiled ceaselessly happy in his chosen work in two spheres which are less widely separated than might at first glance appear. A good citizen, taking on himself more of the burden of the life of his local community than was, perhaps, fair to his wider activities and more universal services. Respected by all who knew him, admired by all who knew him, but above all loved by all who knew him, most of all by those who knew themselves to be his debtors but of whom he would not acknowledge himself a creditor. His life was a revelation of what a " full life " could mean.

There must have been very many of us who rejoiced especially that he could be with us in the celebration by the Society which he served so well — and, the word keeps coming back, so modestly in his greatness — of its hundredth birthday.

R.W.-W.



Charles John Philip CAVE, M.A., F.R.A.S., F.S.A., F.R.P.S., 1871-1950

(President 1913-15, 1924-26)

Plate I.

Figure 2

Figure 3

Figure 4

Figure 5

R. S. Scorer, Billow clouds. Figures 2-4 : Billow clouds proper ; rows of vortices with wave motion above and below. Figure 5. Convection-cell cloud. The cells may sometimes be aligned in rows, but even then no wave motion occurs.

Plate II. (From the Clarke Collection, Royal Meteorological Society).

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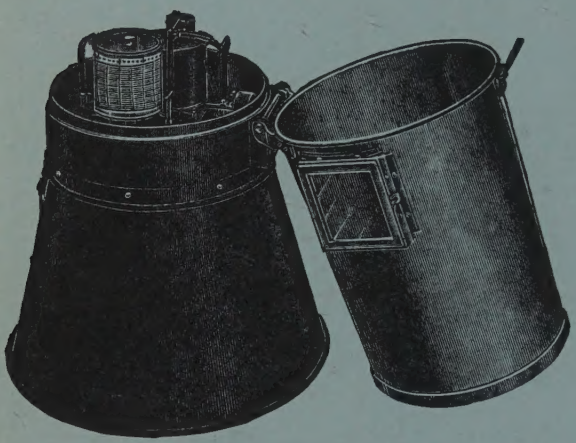

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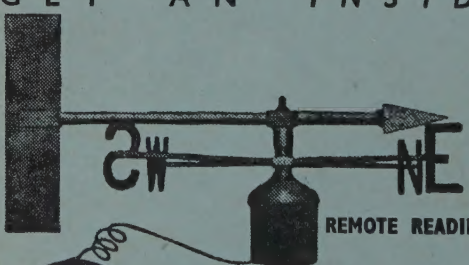

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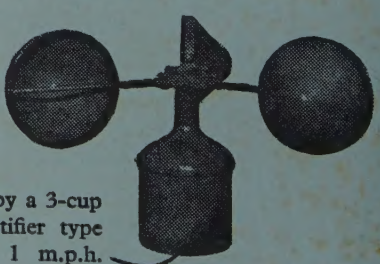

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